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Incorporating Flexible Demand Systems
In Empirical Models of Market Power

By Everett B. Peterson
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Abstract

Measuring the degree of price coordination between firms in a differentiated products industry is particularly challenging because it is necessary to utilize a demand system that is sufficiently flexible, allows the imposition of theoretical restrictions, and allow for the derivation of the functional form of the corresponding price reaction functions. Previous research has relied on restrictive demand systems in order to maintain the tractability of the price reaction functions. The purpose of this paper is determine whether using more flexible demand systems can yield a set of first-order profit maximization conditions that are mathematically tractable and amendable to estimation. The demand systems considered are the Almost Ideal Demand System (AIDS), the Linear Approximate Almost Ideal Demand System (LAIDS), and the Rotterdam demand system. This paper also expands prior work on estimating brand level demand elasticities by endogenizing category level expenditures in the context of a weakly separable demand system. This yields some new and interesting insights for the measurement of market power in differentiated product industries. We show that while it is not possible to derive explicit price reaction functions for any of these demand systems, given certain assumptions, the Rotterdam demand system does yield an explicit set of profit maximization first-order conditions that can be estimated.

1. Introduction

In applied industrial organization research, one common research objective is to measure the degree of price coordination between firms in a differentiated products industry. To estimate the price reaction elasticities (or price conjectures) is particularly challenging because it is necessary to utilize a demand system that is sufficiently flexible, allows the imposition of theoretical restrictions, and allow for the derivation of the functional form of the corresponding price reaction functions (Cotterill, Franklin, and Ma, 1996). Previous research has relied on restrictive demand systems [e.g. Liang (1987) uses a linear demand system] in order to derive an explicit expression for the price reaction functions. The purpose of this paper is determine whether using more flexible demand systems can yield a set of first-order profit maximization conditions that are mathematically tractable and amendable to estimation. The demand systems considered are the Almost Ideal Demand System (AIDS, see Deaton and Muellbauer), the Linear Approximate Almost Ideal Demand System (LAIDS), and the Rotterdam demand system. This paper also expands prior work on estimating brand level demand elasticities by endogenizing category level expenditures in the context of a weakly separable demand system. This yields some new and interesting insights for the measurement of market power in differentiated product industries. We will show that while it is not possible to derive explicit price reaction functions for any of these demand systems, given certain assumptions, the Rotterdam demand system does yield an explicit set of profit maximization first-order conditions that can be estimated. Finally, because of the complexity of these profit maximizing first-order conditions, we also estimate a first-order approximation of a general price reaction function following Cotterill (1998), Cotterill and Putsis (1999) and Cotterill, Putsis, and Dhar 1999, and Putsis (1998). Both models are applied to the ketchup industry. These two approaches are comparable only when firm conjectures are consistent because the former estimates brand level price conjectural elasticities while the latter estimates price reaction elasticities. Liang (1987) has shown that for a linear demand system that brand level models for most, but not all, breakfast cereals do not yield consistent conjectures. In other words, the estimated price conjectures are not equal to the estimate price reaction elasticities.

2. Measuring Brand-Level Price Coordination

This section develops a theoretical model of profit maximization for firms selling a single differentiated brand in an industry with Bertrand price competition (i.e., price is the firm's choice variable). In such as industry, the ability of a firm to price above marginal cost depends on its partial own-price unilateral demand elasticity, how rivals respond to its price change, and the cross-price demand elasticities (Cotterill, 1994). If firms recognize that changes in prices may also induce changes in the level of expenditures on all brands, then this too will affect a single firm's ability to price above marginal cost. This section concludes by specifying an empirical model to measure price coordination between brands.

2.1 General Theoretical Model

Consider the static profit maximization problem facing the *ith* firm, selling a single brand, where price is the firm's choice variable:

$$\max \pi_i = p_i q_i(p_1, \cdots, p_n, X) - c_i(q_i, \mathbf{r_i}) - FC \qquad (1)$$

where p_i is the price of brand i, q_i is the quantity of brand i sold, X is total expenditures on all brands, c_i is the cost function for the ith firm, $\mathbf{r_i}$ is a vector of factor prices facing the *ith* firm, and FC is the level of fixed or sunk costs.

Note that because the demand function posited is a function of brand prices and total brand expenditures, it is assumed that consumer demand for all brands in the industry are weakly separable from all other goods.[1] This assumption is empirically necessary in order to estimate the demand for any good or brand (it's impossible to include the prices of all goods when estimating a demand system). The assumption of weak separability implies a multi-stage budgeting process by consumers where expenditures are allocated to various separable groups based on consumer preferences, relative prices among separable groups, and the level of income. Relative price changes among the separable groups may lead to the reallocation of expenditures among the groups (i.e., the cross-price effect between two goods in different separable groups is shown to be proportional to the income effects for those goods). Thus, changes in price by the *ith* brand may not only illicit a price response by rivals in the industry, it may

[1] Demographic variables may also be included in the brand demand functions. Because demographic variables are generally assumed to be exogenous in demand models, we do not include them in deriving the theoretical and empirical models.

also change the amount of expenditures being spent on all brands. In other words, expenditure in a weakly separable demand system may be endogenous (Brown, *et al*., Capps, *et al*., LaFrance).

Allowing firms to take into account changes in total brand expenditures, the first-order condition for profit maximization for the *ith* firm/brand can be expressed as follows.

$$\frac{\partial \pi_i}{\partial p_i} = q_i(\mathbf{p}, X) + \left(p_i - \frac{\partial c_i}{\partial q_i}\right)\left[\sum_{j=1}^{n}\frac{\partial q_i}{\partial p_j}\frac{\partial p_j}{\partial p_i} + \frac{\partial q_i}{\partial X}\sum_{j=1}^{n}\frac{\partial X}{\partial p_j}\frac{\partial p_j}{\partial p_i}\right] = 0. \quad (2)$$

With some manipulation, equation (2) may be expressed in terms of demand and conjecture elasticities:

$$q_i(\mathbf{p}, X) + \left(p_i - \frac{\partial c_i}{\partial q_i}\right)\left[\sum_{j=1}^{n}\frac{\partial q_i}{\partial p_j}\frac{p_j}{q_i}\frac{\partial p_j}{\partial p_i}\frac{p_i}{p_j}\frac{q_i}{p_i} + \frac{\partial q_i}{\partial X}\frac{X}{q_i}\sum_{j=1}^{n}\frac{\partial X}{\partial p_j}\frac{p_j}{X}\frac{\partial p_j}{\partial p_i}\frac{p_i}{p_j}\frac{q_i}{p_i}\right] = 0$$

$$q_i(\mathbf{p}, X) + \left(p_i - \frac{\partial c_i}{\partial q_i}\right)\frac{q_i}{p_i}\left[\sum_{j=1}^{n}\varepsilon_{ij}\varphi_{ji} + \theta_i\sum_{j=1}^{n}\mu_j\varphi_{ji}\right] = 0. \quad (3)$$

Where ε_{ij} is the uncompensated cross-price elasticity of demand between brands *i* and *j*, holding total brand expenditures constant; φ_{ji} is the conjectured price reaction elasticity between brands *j* and *i;* θ_i is the expenditure elasticity for brand *i*; and μ_j is the group expenditure elasticity for a change in the price of the *jth* brand. (Note: φ_{ii} is equal to 1.) We can define the conjectured price elasticity for brand *i* as:[2]

$$\eta_i^c = \sum_{j=1}^{n}\varepsilon_{ij}\varphi_{ji} + \theta_i\sum_{j=1}^{n}\mu_j\varphi_{ji}. \quad (4)$$

Using equation (4), we can solve for price in equation (3):

$$p_i = \frac{mc_i\eta_i^c}{1+\eta_i^c} \quad (5)$$

where mc_i is the marginal cost for firm *i*. Equation (5) can also be restated as the Lerner index:

$$\frac{p_i - mc_i}{p_i} = -\frac{1}{\eta_i^c} \quad (6)$$

which the familiar first-order condition for profit maximization.

A brand's conjectured price elasticity can be decomposed into two main components. The first component is the brand's unilateral market power. If rivals do not respond to a change in brand price by firm *i*, then the conjectured price elasticity in equation (4) equals the partial own-price demand elasticity plus the expenditure elasticity for brand *i* times the group expenditure elasticity. Note that if brand *i* is a normal good, implying a positive expenditure elasticity, and if an increase in the price of brand *i* leads to an increase in group expenditures on all brands, implying an inelastic aggregate price elasticity, the income effect enhances the firm's unilateral market power. Because previous empirical studies have found inelastic demand for many food products categories, the income effect may play an important role in enhancing the unilateral market power of food manufacturers.

The second component of a brand's conjectured price elasticity measures the affect of price coordination between brands. The first term in equation (4), when *i* is not equal to *j*, looks at the effect of rival reactions on the conjectured price elasticity, holding total group expenditures constant. In a Bertrand pricing game, firms follow each other's price changes, implying that the price reaction elasticities are positive (Deneckere and Davidson). If all brands are substitutes in demand, implying all of the cross-price demand elasticities are positive, then the observed price elasticity is smaller in absolute terms than the own-price demand elasticity. The second term in equation (4) measures the effect of the firm's and rivals' price changes on the level of group expenditures. This term is positive if the brand is a normal good and if total brand expenditures increase as brand prices increase, and the price reaction elasticities are positive. Thus, price coordination may have two distinct ways of enhancing firms' potential market power.

To estimate the coefficients in the price reaction equations given in equation (4), we need to supplement this equation with a set of demand equations, to estimate the own-price, cross-price, and expenditure elasticities of demand, and an equation for group expenditures to estimated the group expenditure elasticities.[3] In general

[2] This conjectured price elasticity depends upon managers' price conjectures, and as such, is different then the observed price elasticity of Cotterill (1994) and Cotterill et al. (1996), Cotterill, Putsis and Dhar (1999). The observed elasticities also called the residual (Baker and Breshnahan) or total elasticities [Tomek and Robinson, Cotterill, Putsis and Dhar (1999)] depends on the estimated price reaction elasticities rather than the estimated conjectural elasticities. As such, it measures the observed changes in prices, not the managers conjectured price changes. The conjectured and observed price elasticities are identical if the managers price conjectures are consistent.

[3] An alternative approach to treating group expenditure as endogenous is to estimate a multi-stage demand system (Hausman 1994; Cotterill and Haller 1997).

notation, the formal model to estimate the price reaction elasticities may be expressed as:

$$q_i = q_i(\mathbf{P}, X), \quad \forall\ i = 1, 2, ..., n, \tag{7}$$

$$p_i = \frac{mc_i \eta_i^c}{1+\eta_i^c}, \quad \forall\ i = 1, 2, ..., n, \text{ and} \tag{8}$$

$$X = X(\mathbf{P}, D). \tag{9}$$

The vector **D** in the expenditure equation represents some vector of exogenous variables, such as per-capita income, income distributions, prices of substitute or complementary goods, and demographic factors that may affect total brand expenditures.

3. Empirical Models

To implement the empirical model given in equations (7) through (9), specific functional forms must be given to the demand equations and the total expenditure equation. The functional forms chosen should be sufficiently flexible to capture a wide variety of consumer behaviors by the model and allow the imposition the theoretical restrictions of homogeneity and symmetry. We consider three commonly used flexible functional forms in applied demand analysis. Namely, the Almost Ideal Demand System (AIDS), the Linear Approximate Almost Ideal Demand System (LA/AIDS), and the Rotterdam demand system.

3.1 Almost Ideal Demand System

The AIDS demand system is specified as:

$$s_i = \alpha_i + \sum_{j=1}^{n}\delta_{ij}\ln p_j + \beta_i\left(\ln x - \alpha_0 - \sum_{j=1}^{n}\alpha_j \ln p_j - \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}\delta_{jk}\ln p_j \ln p_k\right) \tag{10}$$
$$\forall i = 1, 2, \cdots n-1.$$

Because the AIDS model is derived from a logarithmic expenditure function, the dependent variable is budget share rather than quantity. Equation (10) is used to represent equation (7) in the previous section.

Turning to the supply side of the theoretical model, we next derive the price reaction equations by substituting the AIDS demand system in equation (10) into equation (1):

$$\begin{aligned}\max \pi_i &= X s_i - c_i(r, q_i(p, X)) - FC \\ &= X\left(\alpha_i + \sum_{j=1}^{n}\delta_{ij}\ln p_j + \beta_i\left(\ln X - \alpha_0 - \sum_{j=1}^{n}\alpha_j \ln p_j - \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}\delta_{jk}\ln p_j \ln p_k\right)\right) - \\ &\quad c_i(r, q_i(p, X)) - FC.\end{aligned} \tag{11}$$

Differentiating equation (11) with respect to p_i yields the first-order condition for profit maximization:

$$\begin{aligned}\frac{\partial \pi_i}{\partial p_i} = 0 = X\Bigg[&\frac{\delta_{ii}}{p_i} + \sum_{k\neq i}^{n}\delta_{ik}\frac{\partial \ln p_k}{\partial p_i} + \beta_i\Bigg(\sum_{j=1}^{n}\frac{\partial \ln x}{\partial p_j}\frac{\partial p_j}{\partial p_i} - \frac{\alpha_i}{p_i} - \sum_{k\neq i}^{n}\alpha_k\frac{\partial \ln p_k}{\partial p_i} - \\ &\sum_{j=1}^{n}\delta_{ij}\frac{\ln p_j}{p_i} - \sum_{k\neq i}^{n}\sum_{j=1}^{n}\delta_{kj}\ln p_j\frac{\partial \ln p_k}{\partial p_i}\Bigg)\Bigg] + s_i\sum_{j=1}^{n}\frac{\partial X}{\partial p_j}\frac{\partial p_j}{\partial p_i} - \\ &\frac{\partial c_i}{\partial q_i}\left[\sum_{j=1}^{n}\frac{\partial q_i}{\partial p_j}\frac{\partial p_j}{\partial p_i} + \frac{\partial q_i}{\partial X}\sum_{j=1}^{n}\frac{\partial X}{\partial p_j}\frac{\partial p_j}{\partial p_i}\right].\end{aligned} \tag{12}$$

Note that the symmetric conditions of $\delta_{ij} = \delta_{ji}$ have been imposed in equation (12). Multiplying both sides of equation (12) by (p_i/X) yields:

$$\begin{aligned}\frac{\partial \pi_i}{\partial p_i}\frac{p_i}{X} = 0 = \delta_{ii} &+ \sum_{k\neq i}^{n}\delta_{ik}\frac{\partial \ln p_k}{\partial \ln p_i} + \beta_i\Bigg(\sum_{j=1}^{n}\frac{\partial \ln x}{\partial \ln p_j}\frac{\partial \ln p_j}{\partial \ln p_i} - \alpha_i - \sum_{k\neq i}^{n}\alpha_k\frac{\partial \ln p_k}{\partial \ln p_i} - \\ &\sum_{j=1}^{n}\delta_{ij}\ln p_j - \sum_{k\neq i}^{n}\sum_{j=1}^{n}\delta_{kj}\ln p_j\frac{\partial \ln p_k}{\partial \ln p_i}\Bigg) + s_i\sum_{j=1}^{n}\frac{\partial X}{\partial p_j}\frac{p_j}{X}\frac{\partial p_j}{\partial p_i}\frac{p_i}{p_j} - \\ &\frac{\partial c_i}{\partial q_i}\left[\sum_{j=1}^{n}\frac{\partial q_i}{\partial p_j}\frac{p_j}{q_i}\frac{\partial p_j}{\partial p_i}\frac{p_i}{p_j}\frac{q_i}{X} + \frac{\partial q_i}{\partial X}\frac{X}{q_i}\sum_{j=1}^{n}\frac{\partial X}{\partial p_j}\frac{p_j}{X}\frac{\partial p_j}{\partial p_i}\frac{p_i}{p_j}\frac{q_i}{X}\right].\end{aligned} \tag{13}$$

Let: $\frac{\partial \ln X}{\partial \ln p_j} = \frac{\partial X}{\partial p_j}\frac{p_j}{X} = \mu_j$, group expenditure elasticity with respect to price of brand *j*,

$\frac{\partial q_i}{\partial p_j}\frac{p_j}{q_i} = \varepsilon_{ij}$, partial demand elasticity between brands *i* and *j*,

$\frac{\partial \ln p_k}{\partial \ln p_i} = \frac{\partial p_k}{\partial p_i}\frac{p_i}{p_k} = \phi_{ki}$, conjectured price reaction elasticity for brand *k* by firm *i*, and

$\frac{\partial q_i}{\partial X}\frac{X}{q_i} = \theta_i$, expenditure elasticity of demand for brand *i*.

Now, we may rewrite equation (13) as:

$$\begin{aligned}&\sum_{j=1}^{n}\delta_{ij}\phi_{ji} + \beta_i\left(\sum_{j=1}^{n}(\mu_j - \alpha_j)\phi_{ji} - \sum_{j=1}^{n}\delta_{ij}\ln p_j - \sum_{k\neq i}^{n}\sum_{j=1}^{n}\delta_{kj}\ln p_j\phi_{ki}\right) + s_i\sum_{j=1}^{n}\mu_j\phi_{ji} - \\ &\frac{\partial c_i}{\partial q_i}\frac{q_i}{X}\left[\sum_{j=1}^{n}\varepsilon_{ij}\phi_{ji} + \theta_i\sum_{j=1}^{n}\mu_j\phi_{ji}\right] = 0.\end{aligned} \tag{14}$$

Notice that from equation (4), the last term in equation (14) may be expressed as $\frac{\partial c_i}{\partial q_i}\frac{q_i}{X}\eta_i^c$. Rearranging terms in equation (14) to get p_i on the left-hand side of the expression (remember, this is the choice variable for firm *i*) gives:

$$\begin{aligned}\beta_i\left(\sum_{j=1}^{n}\delta_{ij}\phi_{ji}\right)\ln p_i = \sum_{j=1}^{n}\delta_{ij}\phi_{ji} &+ \beta_i\left(\sum_{j=1}^{n}(\mu_j - \alpha_j)\phi_{ji} - \sum_{k\neq i}^{n}\sum_{j=1}^{n}\delta_{kj}\phi_{ji}\ln p_k\right) + \\ &s_i\sum_{j=1}^{n}\mu_j\phi_{ji} - \frac{\partial c_i}{\partial q_i}\frac{q_i}{X}\eta_i^c.\end{aligned} \tag{15}$$

Simplifying equation (15) yields:

$$\ln p_i = \frac{1}{\beta_i} + \frac{\sum_{j=1}^{n}(\mu_j - \alpha_j)\phi_{ji}}{\sum_{j=1}^{n}\delta_{ij}\phi_{ji}} + \frac{s_i\sum_{j=1}^{n}\mu_j\phi_{ji} - \frac{\partial c_i}{\partial q_i}\frac{q_i}{X}\eta_i^c}{\beta_i\sum_{j=1}^{n}\delta_{ij}\phi_{ji}} - \frac{\sum_{k\neq i}^{n}\sum_{j=1}^{n}\delta_{kj}\phi_{ji}\ln p_k}{\sum_{j=1}^{n}\delta_{ij}\phi_{ji}}. \quad (16)$$

Taking a closer look at the term $\frac{\partial c_i}{\partial q_i}\frac{q_i}{X}$ in equation (16), by multiplying by $\frac{c_i}{c_i}\frac{q_i}{q_i}$ yields:

$$\frac{\partial c_i}{\partial q_i}\frac{q_i}{X} = \frac{\partial c_i}{\partial q_i}\frac{q_i}{c_i}\frac{c_i}{q_i}\frac{q_i}{X} = \eta_{vc}\frac{c_i}{X}, \quad (17)$$

where η_{vc} is the total variable cost elasticity. If marginal costs are assumed to be constant with respect to output, then $\eta_{vc}=1$. Using this assumption and multiplying equation (16) by p_iq_i/p_iq_i yields:

$$\frac{c_i}{p_iq_i}\frac{p_iq_i}{x} = \frac{c_i}{p_iq_i}s_i = vc_is_i \quad (18)$$

where vc_i may be interpreted as total variable costs as a percent of sales for brand *i*. Substituting equation (18) into equation (16) gives:

$$\ln p_i = \frac{1}{\beta_i} + \frac{\sum_{j=1}^{n}(\mu_j - \alpha_j)\phi_{ji}}{\sum_{j=1}^{n}\delta_{ij}\phi_{ji}} + \frac{s_i\left(\sum_{j=1}^{n}\mu_j\phi_{ji} - vc_i\eta_i^c\right)}{\beta_i\sum_{j=1}^{n}\delta_{ij}\phi_{ji}} - \frac{\sum_{k\neq i}^{n}\sum_{j=1}^{n}\delta_{kj}\phi_{ji}\ln p_k}{\sum_{j=1}^{n}\delta_{ij}\phi_{ji}}. \quad (19)$$

However, equation (19) is not yet a reaction function for firm *i* because μ_j, s_i, vc_i, and η_i^c are, in general, functions of all brand prices. If a log-linear functional form is chosen to represent group expenditures in equation (9), then we may treat μ_j as a constant. However, substituting for s_i and η_i^c in equation (19) makes it impossible to solve for the reaction functions because the share equations are quadratic in logarithm of prices.

If one retreats from explicitly deriving the price reaction function, it still may be possible to estimate the conjectural elasticities. Since, equation (19) represents the first-order condition for profit maximization for the *i*th firm facing an AIDS demand function, one alternative may be to estimate equation (19) directly and obtain the estimates of the conjectured price reaction elasticities (ϕ_{ji}). From an econometric perspective, because the share of brand *i* is an endogenous variable in the AIDS demand system, one may be able to avoid substituting for s_i in equation (19). However, this approach is not likely to be successful for several reasons. First, information is required on firm variable costs as a percent of sales, which is generally not available, and the conjectured price elasticity (η_i^c) is itself a function of prices. Second, given the non-linear nature of the equation (19), one may not be able to identify and thus estimate all of the conjectured price reaction elasticities (ϕ_{ji}).

3.2 Linear Approximate Almost Ideal Demand System

Because of the complex form of the firm price reaction functions derived from the nonlinear AIDS demand system, the Linear Almost Ideal Demand System (LA/AIDS) may be more mathematically tractable than the original AIDS specification. The LA/AIDS model is specified as:

$$s_i = \alpha_i + \sum_{j=1}^{n}\delta_{ij}\ln p_j + \beta_i\left(\ln X - \sum_{j=1}^{n}s_j^0\ln p_j\right), \quad \forall\, i = 1,\ldots,n-1. \quad (20)$$

Note that the term $\sum_{j=1}^{n}s_j^0\ln p_j$ is a log-linear analogue of the Laspeyres index that uses base shares (s_j^0) and prices (p_j) that are scaled by their means.[4]

Substituting equation (20) into the objective function given in equation (1) gives:

$$\max \pi_i = X\left(\alpha_i + \sum_{j=1}^{n}\delta_{ij}\ln p_j + \beta_i\left(\ln X - \sum_{j=1}^{n}s_j^0\ln p_j\right)\right) - c_i(r, q_i(P,X)) - FC \quad (21)$$

with respect to p_i. The first-order condition for profit maximization is:

$$\frac{\partial \pi_i}{\partial p_i} = X\left[\frac{\delta_{ii}}{p_i} + \sum_{k\neq i}^{n}\delta_{ik}\frac{\partial \ln p_k}{\partial p_i} + \beta_i\sum_{j=1}^{n}\frac{\partial \ln X}{\partial p_j}\frac{\partial p_j}{\partial p_i} - \beta_i\left(\frac{s_i^0}{p_i} + \sum_{k\neq i}^{n}s_k^0\frac{\partial \ln p_k}{\partial p_i}\right)\right] + \quad (22)$$
$$\left(\alpha_i + \sum_{j=1}^{n}\delta_{ij}\ln p_j + \beta_i\left(\ln X - \sum_{j=1}^{n}s_j^0\ln p_j\right)\right)\sum_{j=1}^{n}\frac{\partial X}{\partial p_j}\frac{\partial p_j}{\partial p_i} -$$
$$\frac{\partial c_i}{\partial q_i}\left[\sum_{j=1}^{n}\frac{\partial q_i}{\partial p_j}\frac{\partial p_j}{\partial p_i} + \frac{\partial q_i}{\partial X}\sum_{j=1}^{n}\frac{\partial X}{\partial p_j}\frac{\partial p_j}{\partial p_i}\right] = 0$$

Multiply both sides of equation (22) by (p_i/X) yields:

[4] Moschini has shown that the Stone index is not invariant to units of measurement for the prices. The log-linear analogue to the Laspreyes index is one of the indices Moschini suggests to solve the invariance problem.

$$\frac{\partial \pi_i}{\partial p_i}\frac{p_i}{X} = \delta_{ii} + \sum_{k\neq i}^{n}\delta_{ik}\frac{\partial \ln p_k}{\partial \ln p_i} + \beta_i\sum_{j=1}^{n}\frac{\partial \ln X}{\partial \ln p_j}\frac{\partial \ln p_j}{\partial \ln p_i} - \beta_i\left(s_i^0 + \sum_{k\neq i}^{n} s_k^0\frac{\partial \ln p_k}{\partial \ln p_i}\right) + \left(\alpha_i + \sum_{j=1}^{n}\delta_{ij}\ln p_j + \beta_i\left(\ln X - \sum_{j=1}^{n}s_j^0 \ln p_j\right)\right)\sum_{j=1}^{n}\frac{\partial X}{\partial p_j}\frac{p_j}{X}\frac{\partial p_j}{\partial p_i}\frac{p_i}{p_j} - \frac{\partial c_i}{\partial q_i}\left[\sum_{j=1}^{n}\frac{\partial q_i}{\partial p_j}\frac{p_j}{q_i}\frac{\partial p_j}{\partial p_i}\frac{p_i}{p_j}\frac{q_i}{X} + \frac{\partial q_i}{\partial X}\frac{X}{q_i}\sum_{j=1}^{n}\frac{\partial X}{\partial p_j}\frac{p_j}{X}\frac{\partial p_j}{\partial p_i}\frac{p_i}{p_j}\frac{q_i}{X}\right] = 0. \quad (23)$$

Let:

$\frac{\partial \ln X}{\partial \ln p_j} = \frac{\partial X}{\partial p_j}\frac{p_j}{X} = \mu_j$, group expenditure elasticity with respect to price of brand *j*,

$\frac{\partial q_i}{\partial p_j}\frac{p_j}{q_i} = \varepsilon_{ij}$, partial demand elasticity between brands *i* and *j*,

$\frac{\partial \ln p_k}{\partial \ln p_i} = \frac{\partial p_k}{\partial p_i}\frac{p_i}{p_k} = \phi_{ki}$, conjectured price reaction elasticity between brands *k* and *i*, and

$\frac{\partial q_i}{\partial X}\frac{X}{q_i} = \eta_i$, expenditure elasticity of demand for brand *i*.

Now, equation (23) may be written as:

$$\sum_{j=1}^{n}\delta_{ij}\phi_{ji} + \beta_i\sum_{j=1}^{n}\left(\mu_j - s_j^0\right)\phi_{ji} + s_i\sum_{j=1}^{n}\mu_j\phi_{ji} - \frac{\partial c_i}{\partial q_i}\frac{q_i}{X}\left[\sum_{j=1}^{n}\varepsilon_{ij}\phi_{ji} + \eta_i\sum_{j=1}^{n}\mu_j\phi_{ji}\right] = 0. \quad (24)$$

At the point of normalization (e.g., where all prices are normalized to unity) and when α_0 in the AIDS functional form in equation (10) is set equal to expenditure in a base period (or if expenditures are normalized to equal one) such that the α_i equal the predicted budget shares, then Asche and Wessells show that the uncompensated demand elasticity for the AIDS model can be expressed as:

$$\varepsilon_{ij} = -\Delta_{ij} + \frac{\delta_{ij} - \beta_i s_j}{s_i}, \quad (25)$$

where Δ_{ij} = 1 for all $i = j$, and zero otherwise. This expression is identical to the formulas used by Chalfant and Buse at the point of normalization. In addition, the expenditure elasticity for the AIDS model is defined as:

$$\eta_i = \frac{\beta_i}{s_i} + 1. \quad (26)$$

Asche and Wessells also show that this expression is appropriate for the LA/AIDS at the point of normalization.

Substituting equations (25) and (26) into equation (24) yields:

$$\sum_{j=1}^{n}\delta_{ij}\phi_{ji} + \beta_i\sum_{j=1}^{n}\left(\mu_j - s_j^0\right)\phi_{ji} + s_i\sum_{j=1}^{n}\mu_j\phi_{ji} - \frac{\partial c_i}{\partial q_i}\frac{q_i}{X}\left[\frac{-s_i + \sum_{j=1}^{n}\delta_{ij}\phi_{ji} + \beta_i\sum_{j=1}^{n}\left(\mu_j - s_j^0\right) + s_i\sum_{j=1}^{n}\mu_j\phi_{ji}}{s_i}\right] = 0. \quad (27)$$

Using equations (17) and (18), we may rewrite equation (27) as:

$$\left(1 - vc_i\right)\left[\sum_{j=1}^{n}\delta_{ij}\phi_{ji} + \beta_i\sum_{j=1}^{n}\left(\mu_j - s_j^0\right)\phi_{ji} + s_i\sum_{j=1}^{n}\mu_j\phi_{ji}\right] + vc_i s_i = 0. \quad (28)$$

Now, one can solve for s_i in equation (28):

$$s_i = \frac{\left(vc_i - 1\right)\left[\sum_{j=1}^{n}\left(\delta_{ij} + \beta_i\left(\mu_j - s_j^0\right)\right)\phi_{ji}\right]}{\left(1 - vc_i\right)\sum_{j=1}^{n}\mu_j\phi_{ji} + vc_i}. \quad (29)$$

But, from equation (20), s_i is a function of all prices. So substituting equation (20) into equation (29) and solving for the logarithm of p_i gives the price reaction equation for brand *i*:

$$\alpha_i + \sum_{j=1}^{n}\delta_{ij}\ln p_j + \beta_i\left(\ln X - \sum_{j=1}^{n}s_j^0\ln p_j\right) = \frac{\left(vc_i - 1\right)\left[\sum_{j=1}^{n}\left(\delta_{ij} + \beta_i\left(\mu_j - s_j^0\right)\right)\phi_{ji}\right]}{\left(1 - vc_i\right)\sum_{j=1}^{n}\mu_j\phi_{ji} + vc_i}$$

Collecting terms yields:

$$\ln p_i = \frac{A_i - \alpha_i}{\delta_{ii} - \beta_i s_i^0} - \frac{\sum_{j\neq i}^{n}\left(\delta_{ij} - \beta_i s_j^0\right)\ln p_j}{\delta_{ii} - \beta_i s_i^0} - \frac{\beta_i \ln X}{\delta_{ii} - \beta_i s_i^0}, \quad (30)$$

where:

$$A_i = \frac{\left(vc_i - 1\right)\left[\sum_{j=1}^{n}\left(\delta_{ij} + \beta_i\left(\mu_j - s_j^0\right)\right)\phi_{ji}\right]}{\left(1 - vc_i\right)\sum_{j=1}^{n}\mu_j\phi_{ji} + vc_i}.$$

Strictly speaking, equation (30) is only a reaction function if vc_i and X are not functions of brand prices. Even if one wishes to directly estimate equation (30) as part of a system of equations [including equations (9) and (20)], it may not be possible to obtain information on vc_i. Without this information, it would be impossible to identify the conjectural elasticities in the term A_i in equation (30).

3.3 Rotterdam Demand System

The last demand system considered is the Rotterdam model. But because the Rotterdam model is specified as changes in prices, quantities, and expenditure, we will need to re-specify equations (7) through (9) in differential logarithmic form, rather than in levels.

$$\bar{s}_i\, d\ln q_i = \alpha_i\left(d\ln X - \sum_{j=1}^{n}\bar{s}_j\, d\ln p_j\right) + \sum_{j=1}^{n}\delta_{ij}\, d\ln p_j,\ \forall i = 1,2,\ldots,n-1, \quad (31)$$

$$d\ \ln\ p_i = d\ \ln mc_i + d\ \ln \tau_i,\ \forall\, i = 1, \ldots, n\text{, and} \quad (32)$$

$$d\ln X = \sum_{j=1}^{n}\mu_j\, d\ln p_j + \sum_{k=1}^{m}\beta_k\, d\ln p_k^s + \rho\ d\ln I + \sum_{h=1}^{s}\gamma_h\, d\ln D_h, \quad (33)$$

where $\bar{s}_i = \dfrac{s_{i,t} + s_{i,\,t-1}}{2}$ is the average budget share, $\tau_i = \dfrac{\eta_i^c}{1+\eta_i^c}$, p_k^s is the price of other substitute goods, I is per-capita income, and D_h is a vector of demographic variables . All differentials are implemented as first-difference approximations.

The term $d\ln\tau_i$ in equation (32) requires some additional attention, because as seen from equation (4), η_i^c may be a function of prices. Using the definitions of the price and expenditure elasticities for the Rotterdam model,

$$\varepsilon_{ij} = \frac{\delta_{ij} - s_j\alpha_i}{s_i},\ \text{and}$$

$$\theta_i = \frac{\alpha_i}{s_i}$$

equation (4) can be rewritten as:

$$\eta_i^c = \frac{1}{s_i}\sum_{j=1}^{n}\left[\delta_{ij} + \alpha_i\left(\mu_j - s_j\right)\right]\phi_{ji}. \quad (34)$$

Using equation (34), we can define τ_i in equation (32) as:

$$\tau_i = \frac{\frac{1}{s_i}\sum_{j=1}^{n}\left[\delta_{ij} + \alpha_i\left(\mu_j - s_j\right)\right]\phi_{ji}}{1 + \frac{1}{s_i}\sum_{j=1}^{n}\left[\delta_{ij} + \alpha_i\left(\mu_j - s_j\right)\right]\phi_{ji}} = \frac{\sum_{j=1}^{n}\left[\delta_{ij} + \alpha_i\left(\mu_j - s_j\right)\right]\phi_{ji}}{s_i + \sum_{j=1}^{n}\left[\delta_{ij} + \alpha_i\left(\mu_j - s_j\right)\right]\phi_{ji}}. \quad (35)$$

Taking the logarithm of equation (35) yields:

$$\ln\tau_i = \ln\left\{\sum_{j=1}^{n}\left[\delta_{ij} + \alpha_i\left(\mu_j - s_j\right)\right]\phi_{ji}\right\} - \ln\left\{s_i + \sum_{j=1}^{n}\left[\delta_{ij} + \alpha_i\left(\mu_j - s_j\right)\right]\phi_{ji}\right\}. \quad (36)$$

Because all s_j's are a function of price, take the first-order differential of equation (36) with respect to all s_j in equation (36). (Note, the conjectured price reaction elasticities are treated as parameters that we can hopefully identify.)

$$d\ln\tau_i = -\frac{\alpha_i\sum_{j=1}^{n}\phi_{ji}\, d\, s_j}{\sum_{j=1}^{n}\left[\delta_{ij} + \alpha_i\left(\mu_j - s_j\right)\right]\phi_{ji}} - \frac{ds_i - \alpha_i\sum_{j=1}^{n}\phi_{ji}\, d\, s_j}{s_i + \sum_{j=1}^{n}\left[\delta_{ij} + \alpha_i\left(\mu_j - s_j\right)\right]\phi_{ji}} \quad (37)$$

$$= -\frac{\alpha_i\sum_{j=1}^{n}\phi_{ji}\, d\, s_j}{s_i\eta_i^c} - \frac{d\, s_i - \alpha_i\sum_{j=1}^{n}\phi_{ji}\, d\, s_j}{s_i\left(1+\eta_i^c\right)}.$$

Combining terms in equation (37) and noting that ϕ_{ii} equals 1 yields:

$$d\ln\tau_i = -\frac{1}{s_i}\left[\alpha_i\sum_{j=1}^{n}\phi_{ji}\, d\, s_j\left(\frac{1}{\eta_i^c} - \frac{1}{1+\eta_i^c}\right) + \frac{d\, s_i}{1+\eta_i^c}\right] \quad (38)$$

$$= -\frac{1}{s_i\eta_i^c\left(1+\eta_i^c\right)}\left[\left(\alpha_i + \eta_i^c\right)d\, s_i + \alpha_i\sum_{j\neq i}^{n}\phi_{ji}\, d\, s_j\right].$$

However, since the price terms in the Rotterdam demand equations and the group expenditure equation are in terms of $d\ln p_j$, one would like to get equation (38) in terms of $d\ln s_j$. This can be accomplished using the chain rule of differentiation:

$$\frac{\partial(\cdot)}{\partial s_j}\frac{\partial s_j}{\partial\ln s_j} = \frac{\partial(\cdot)}{\partial s_j}s_j\,.$$

Thus, one can rewrite equation (38) as:

$$d\ln\tau_i = -\frac{1}{s_i\eta_i^c\left(1+\eta_i^c\right)}\left[\left(\alpha_i + \eta_i^c\right)s_i\, d\ln s_i + \alpha_i\sum_{j\neq i}^{n}s_j\phi_{ji}\, d\ln s_j\right]. \quad (39)$$

Next, to derive the expression for $d\ln s_j$, take the logarithm of the budget share:

$$\ln s_j = \ln p_j + \ln q_j - \ln x. \quad (40)$$

Taking the first-order differential of equation (40) yields:

$$d\ln s_j = d\ln p_j + d\ln q_j - d\ln x. \quad (41)$$

Substituting equation (41) into equation (39) yields:

$$d\ln\tau_i = -\frac{1}{s_i\eta_i^c\left(1+\eta_i^c\right)}\left[\left(\alpha_i + \eta_i^c\right)s_i\left(d\ln p_i + d\ln q_i - d\ln x\right) + \alpha_i\sum_{j\neq i}^{n}s_j\phi_{ji}\left(d\ln p_j + d\ln q_j - d\ln x\right)\right]. \quad (42)$$

Using the following definitions:

$$\lambda_{ii} = -\frac{\alpha_i + \eta_i^c}{\eta_i^c\left(1+\eta_i^c\right)}\,, \tag{43}$$

$$\lambda_{ij} = -\frac{\alpha_i s_j \phi_{ji}}{s_i \eta_i^c\left(1+\eta_i^c\right)}\,, \forall\ j \neq i\,, \tag{44}$$

Substituting equations (43) and (44) into equation (42) yields:

$$d\ \ln \tau_i = \sum_{j=1}^{n} \lambda_{ij}\left(d\ \ln\ p_j + d\ \ln\ q_j\right) - d\ \ln\ x \sum_{j=1}^{n} \lambda_{ij}\ . \tag{45}$$

Now one can substitute equation (45) into equation (32) and solve for $d\ln p_i$:

$$d\ln p_i = \frac{1}{\left(1-\lambda_{ii}\right)}\left(d\ln mc_i + \sum_{j\neq i}^{n}\lambda_{ij}\, d\ln p_j + \sum_{j=1}^{n}\lambda_{ij}\, d\ln q_j\right) - d\ln x \frac{\sum_{j=1}^{n}\lambda_{ij}}{\left(1-\lambda_{ii}\right)}. \tag{46}$$

To finish this specification, one needs to determine an expression for $d\ln mc_i$. Begin with a general logarithmic expression for marginal cost:

$$\ln mc_i = mc_i\,(\ln r, \ln q_i), \tag{47}$$

where r is a vector of input prices. Taking the first-order differential of equation (47) yields:

$$d\ \ln mc_i = \sum_{z=1}^{t} \frac{\partial \ln mc_i}{\partial \ln r_z}\, d\ \ln\ r_z + \frac{\partial \ln mc_i}{\partial \ln q_i}\, d\ \ln\ q_i\,. \tag{48}$$

Let: $\frac{\partial \ln mc_i}{\partial \ln r_z} = \nu_{iz} =$ marginal cost elasticity with respect to the *zth* input price and

$\frac{\partial \ln mc_i}{\partial \ln q_i} = \eta_{mc}^{i} =$ marginal cost elasticity with respect to output.

Now equation (47) may be rewritten as:

$$d\ln p_i = \frac{1}{\left(1-\lambda_{ii}\right)}\left(\sum_{z=1}^{t}\nu_{iz}\, d\ln r_z + \left(\eta_{mc}^{i} + \lambda_{ii}\right) d\ln q_i\right) + \sum_{j\neq i}^{n}\frac{\lambda_{ij}}{1-\lambda_{ii}}\left(d\ln p_j + d\ln q_j\right) - \sum_{j=1}^{n}\frac{\lambda_{ij}}{1-\lambda_{ii}}\, d\ln x. \tag{49}$$

Note that equation (49) is not a true price reaction function because it contains the changes in quantities, $d\ln q_j$, and total brand expenditures, $d\ln x$, which are both functions of brand prices. However, as stated above, if the primary goal is to estimate the conjectural elasticities, substituting in expressions for $d\ln q_j$, and $d\ln x$ would only increase the complexity of the model specification.

The brand price coordination model using the Rotterdam demand system includes equations (31), (33), and (49). However, note there are $2n$ equations [(n-1) demand equations from (31), n first-order conditions from equation (49) and 1 equation from (33)] in $2n$+1 unknowns (n $d\ln q_j$, n $d\ln p_j$, and $d\ln x$). The problem arises because one can only estimate (n-1) of the Rotterdam demand equations due to the adding-up condition, but the first-order profit maximization conditions in equation (49) includes all n $d\ln q_j$ variables. Thus, one of the $d\ln q_j$ needs to be eliminated from equation (49). To do so, rewrite equation (49) using equation (40):

$$\begin{aligned} d\ln p_i &= \frac{1}{\left(1-\lambda_{ii}\right)}\left(\sum_{z=1}^{t}\nu_{iz}\, d\ln r_z + \left(\eta_{mc}^{i} + \lambda_{ii}\right) d\ln q_i\right) + \\ &\quad \sum_{j\neq i}^{n}\frac{\lambda_{ij}}{1-\lambda_{ii}}\left(d\ln p_j + d\ln q_j - d\ln x\right) - \frac{\lambda_{ii}}{1-\lambda_{ii}}\, d\ln x \\ &= \frac{1}{\left(1-\lambda_{ii}\right)}\left(\sum_{z=1}^{t}\nu_{iz}\, d\ln r_z + \left(\eta_{mc}^{i} + \lambda_{ii}\right) d\ln q_i\right) + \sum_{j\neq i}^{n}\frac{\lambda_{ij}}{1-\lambda_{ii}}\, d\ln s_j - \frac{\lambda_{ii}}{1-\lambda_{ii}}\, d\ln x. \end{aligned} \tag{50}$$

Because the budget shares must sum to one, one can specify the budget share of the *jth* good as:

$$s_j = 1 - \sum_{k\neq j}^{n} s_k\,. \tag{51}$$

Taking the logarithm of equation (51) yields:

$$\ln s_j = \ln\left(1 - \sum_{k\neq j}^{n} s_k\right). \tag{52}$$

Next, take the first-order differential of equation (52):

$$d\ \ln\ s_j = -\frac{\sum_{k\neq j}^{n} s_k\, d\ \ln\ s_k}{1 - \sum_{k\neq j}^{n} s_k} = -\frac{\sum_{k\neq j}^{n} s_k\, d\ \ln\ s_k}{s_j}. \tag{53}$$

Substituting equation (53) into equation (40) yields:

$$\begin{aligned} d\ln p_j + d\ln q_j - d\ln x &= -\frac{\sum_{k\neq j}^{n} s_k\, d\ln s_k}{s_j} \\ &= -\frac{\sum_{k\neq j}^{n} s_k}{s_j}\left(d\ln p_k + d\ln q_k - d\ln x\right). \end{aligned} \tag{54}$$

Solving for $d\ln q_j$ in equation (54) gives:

$$d\ln q_j = -\sum_{k \neq j}^{n} \frac{s_k}{s_j}\left(d\ln p_k - d\ln q_k\right) - d\ln p_j + \frac{1}{s_j}\, d\ln x. \quad (55)$$

Now, the expression for $d\ln q_j$ may be substituted into the first-order profit maximization condition for each firm. Substituting equation (55) into equation (49) for firm *i*:

$$d\ln p_i = \frac{1}{(1-\lambda_{ii})}\left(\sum_{z=1}^{t} \nu_{iz}\, d\ln r_z + \left(\eta_{mc}^{i} + \lambda_{ii}\right) d\ln q_i\right) + \sum_{k \neq j \neq i}^{n} \frac{\lambda_{ik}}{1-\lambda_{ii}}\left(d\ln p_k + d\ln q_k\right)$$
$$- \sum_{j=1}^{n} \frac{\lambda_{ij}}{1-\lambda_{ii}}\, d\ln x + \frac{\lambda_{ij}}{1-\lambda_{ii}}\left(-\sum_{k \neq j}^{n} \frac{s_k}{s_j}\left(d\ln p_k + d\ln q_k\right) - d\ln p_j + \frac{1}{s_j}\, d\ln x\right) \quad (56)$$

Because there is a $d\ln p_i$ term on both sides of equation (56), solving for $d\ln p_i$ yields:

$$d\ln p_i = \frac{1}{(1-\lambda_{ii})s_j + s_i\lambda_{ij}}\left\{ s_j \sum_{z=1}^{t} \nu_{iz}\, d\ln r_z + \left[s_j\left(\eta_{mc}^{i} + \lambda_{ii}\right) - \lambda_{ij} s_i\right] d\ln q_i + \right.$$
$$\left. \sum_{k \neq j \neq i}^{n} \left(s_j \lambda_{ik} - s_k \lambda_{ij}\right)\left(d\ln p_k + d\ln q_k\right) + \left(\lambda_{ij} - s_j \sum_{m=1}^{n} \lambda_{im}\right) d\ln x \right\}, \forall i \neq j. \quad (57)$$

Notice that in addition to eliminating the variable $d\ln q_j$ from equation (57), the change in the logarithm of the price of the *j*th brand ($d\ln p_j$) has also been eliminated.

Using equation (57), $d\ln q_j$ is eliminated from the first (n-1) profit maximization conditions in equation (49). However, one must also eliminate $d\ln q_j$ from the first-order condition of the *j*th firm:

$$d\ln p_j = \frac{1}{\left(1-\lambda_{jj}\right)}\left[\sum_{z=1}^{t} \nu_{jz}\, d\ln r_z + \left(\eta_{mc}^{j} + \lambda_{jj}\right)\left(-\sum_{k \neq j}^{n} \frac{s_k}{s_j}\left(d\ln p_k + d\ln q_k\right) - d\ln p_j + \right.\right.$$
$$\left.\left. \frac{1}{s_j}\, d\ln x\right)\right] + \sum_{i \neq j}^{n} \frac{\lambda_{ji}}{1-\lambda_{jj}}\left(d\ln p_i + d\ln q_i\right) - \sum_{m=1}^{n} \frac{\lambda_{jm}}{1-\lambda_{jj}}\, d\ln x. \quad (58)$$

Again, because $d\ln p_j$ appears on both sides of equation (58), one can rewrite this equation as:

$$d\ln p_j = \frac{1}{\left(1+\eta_{mc}^{j}\right)} \sum_{z=1}^{t} \nu_{jz}\, d\ln r_z + \frac{1}{\left(1+\eta_{mc}^{j}\right)s_j}\left\{ \sum_{k \neq j}^{n} \left(s_j \lambda_{jk} - s_k\left(\eta_{mc}^{j} + \lambda_{jj}\right)\right) \right. \quad (59)$$
$$\left. \left(d\ln p_k + d\ln q_k\right) + \left(\eta_{mc}^{j} + \lambda_{jj} - s_j \sum_{i=1}^{n} \lambda_{ji}\right) d\ln x \right\}.$$

The final empirical model specification using the Rotterdam demand system is given by equations (31), (33), (57), and (59). Assuming that it is possible to identify all of the model parameters in equations (57) and (59), one is able to retrieve estimates of η_i^c and ϕ_{ji} from estimates of all λ_{ij}. Using equation (43), if one knows the value of λ_{ii} and α_i (which is identified in the Rotterdam demand equations), then one may solve for η_i^c. Once the value of η_i^c has been determined, then one may solve for ϕ_{ji} using equation (44) using estimated values of α_i and observed market shares (s_j). The main drawback of this identification procedure is that λ_{ii} is estimated as a constant parameter, implying that η_i^c is also constant across all time periods and regional markets. Since η_i^c is a function of the price, expenditure, and conjectural elasticities, all of which may take on different values as prices and expenditure change, this implies that these elasticities change in such a manner to ensure η_i^c remains constant. A very strong assumption to make. However, given the non-linear nature of equations (57) and (59) as well as a limited number of data points, it may not be possible to allow λ_{ii} to vary across time and regional markets.

4. Application and Data

In this section, we use the Rotterdam demand system derived above to analyze ketchup brand price coordination. The ketchup industry was chosen because the top three brands, Heinz, Hunts, and Del Monte accounted for approximately 85 percent of all ketchup sales in 42 regional markets in the U.S. between the years 1988 and 1992. The small number of brands helps to reduce the dimensionality of this analysis. In addition, Haller and Cotterill have found evidence of market power in the ketchup industries using traditional brand share–brand price and residual demand measures.

Quarterly data from 1988 to 1992 on dollar value of retail brand sales, average retail unit price of branded products, dollar value of retail private label sales, average retail unit price of private label products for 42 regional markets were obtained from the Information Resources Incorporated (IRI) Info Scan data base. In addition, the IRI data includes information on units per volume (U/VOL), and the magnitude and intensity of merchandising performed by retailers for each brand and all private label products.

The base unit for ketchup in the IRI data is one pound (16 ounces). Because ketchup comes in a variety of sizes, and the average price per unit typically varies with the size of the container, the variable U/VOL is used to control for change in average price due to a change in the mix of container size sold during the quarter. For example, if relatively more 64-ounce bottles of Heinz ketchup are sold from one quarter to the next, the variable U/VOL will increase. Since it likely cost less per ounce to produce a 64-ounce bottle of ketchup than a 16-ounce bottle, the change in U/VOL should be negatively related to the change in brand price.

Because the IRI price data is obtained at the retail level, we need to control for any retailer merchandising activities that may affect the average retail product price.[5] We control for the size of any promotional price reductions and the volume of the product sold during promotional campaigns, because they will likely have differential effects on the average retail price. The retailer merchandising activities are measured by the average price reduction of products merchandised (PRED) and the percent of volume sold with merchandising (%MER). An increase in either or both of these will lead to a lower average retail price, all else constant. Also, the amount of merchandising may also affect the level of ketchup expenditures and relative market shares among ketchup brands.

The IRI data does not contain any demographic information (e.g., median household income, age, ethnic background, etc.) for the regional markets. It is likely that changes in demographic profiles across regional markets may yield differences in demand for ketchup brands and the level of expenditures allocated to ketchup. For example, do ketchup expenditures increase or decrease as median household income increases? Do regions with relatively growing Hispanic or other ethnic groups exhibit changes in ketchup expenditures and/or the mix of ketchup brands purchased? To supplement the IRI data set, we include data on median household income (INC), percent of households with less than $10,000 in income (IU10K), percentage of households with over $50,000 in income (IO50K), percentage of population that is Hispanic (PHSP), median family age (AGE), median family size (SIZE), and the four firm concentration ratio of all grocery stores in each regional market (CR4).[6] The last variable is included to account for possible price enhancement from high retail concentration in local markets.

Earlier, we stated that relative price changes among separable groups may lead to the reallocation of expenditures among them. To control for possibility, we identify five IRI product categories that may be close substitutes or complements with ketchup: mustard, mayonnaise and other sandwich spreads, barbecue sauces, hot sauces, and steak and Worcestershire sauce. We utilize the average prices of all products in these IRI categories in our model.

Finally, equations (57) and (59) identifies that changes in input prices affect brand price changes. However, obtaining a complete list of input prices for ketchup manufacturers is not possible. To attempt to control for changes in input costs, we include the prices of two main ingredients in ketchup, tomato paste (TP) and sweeteners (high fructose corn syrup) (SWT). These data were obtained from industry and U.S. Department of Agriculture publications.

5. Results

5.1 Conjectures Model using Rotterdam Demand System

Table 1 provides the specification for each equation in the ketchup brand-pricing model. An iterative three-stage least squares procedure is used to estimate this system of equations.

Table 2 provides the parameter estimates for the demand and expenditure equations of the model. In the demand equations, all of the estimated real expenditure and price parameters are significantly different than zero and have the correct sign. The resulting expenditure and partial price elasticities for each brand are given in the top half of table 3. The term partial is used for the price elasticities because they are computed holding total ketchup expenditures constant. In absolute value, Heinz and Hunts have the smallest partial own-price elasticities, -1.43 and -1.32 respectively, followed private label ketchup, (-1.84) and then Del Monte (-2.07). This is not surprising since Heinz has the largest market share and Del Monte has the lowest market share and, all else equal, the absolute value of the own-price elasticities decrease as share increases. It is interesting to note that the absolute value of the private label own-price elasticity is similar to that of Hunts. Private label products have been characterized in previous research (Connor and Peterson) as being more competitive than their national brand counterparts. This would suggest that the absolute value of the own-price demand elasticities facing private label producers would be much higher than for the national brands. At least for ketchup, this does not seem to be the case.

Because ketchup is assumed to be part of a weakly separable demand system, changes in brand prices may also affect the amount of total expenditures allocated to ketchup by consumers. Indeed, the estimates from the expenditure equation show that a one percent increase in the price of Heinz decreases total ketchup expenditures by 0.43 percent. Price changes by Hunts, Del Monte, and private label ketchup did not have significant effects

[5] Often, the manufacturers coordinate the merchandising activities of retailers. For example, the manufacturer may wish to have the retailer feature their product and give the retailer a discount on all product sold during a given time period.

[6] These data were obtained from Progressive Grocery, Market Scope, various years. Unfortunately, these data are available on an annual basis only.

on ketchup expenditures. This is likely a reflection of Heinz's relatively large market share in most of the local markets in our sample. Thus, consumers react to a price increase by the market leader by reducing expenditures on ketchup while they seem to ignore price changes by brands with smaller market shares. Using this information, one may compute the "total" brand price elasticities of demand, or as stated earlier, a brand's unilateral market power, as follows:

$$\varepsilon_{ij}^{T} = \varepsilon_{ij} + \theta_i \, \mu_j \, ,$$

where ε_{ij}^{T} is the total price elasticity between brands *i* and *j*, ε_{ij}, is the partial uncompensated price elasticity, θ_i, is the expenditure elasticity for the *i*th brand, and μ_j is the group expenditure elasticity with respect to a change in the *j*th brand price. The bottom half of table 3 contains the computed total elasticities. Because of its negative impact on total ketchup expenditure, Heinz's unilateral market power is virtually the same as Del Monte and private label ketchup. Thus, in this case, the income effect does not enhance Heinz's unilateral market power, relative to the other ketchup brands.

The parameter estimates in the expenditure equation also provide information on the impacts of price changes of related goods and household income on ketchup expenditures. Mustard and mayonnaise are found to be complements to ketchup, with increases in their average prices decreasing ketchup expenditures. Conversely, barbecue and steak sauces are substitutes to ketchup, with steak sauce being a very strong substitute (a one percent increase in the price of steak sauce increases ketchup expenditures by 1.5 percent). While all of the brand expenditure elasticities are positive, increases in median household income reduce ketchup expenditures. Thus, not surprisingly, consumers view ketchup as an inferior good.

Retailer promotional activities for ketchup affect both the relative brand market shares and the level of ketchup expenditures. Table 4 lists the own and cross merchandising elasticities for all brands, holding total expenditures constant and allowing for merchandising to change the level of expenditures. When ketchup expenditures are held constant, increases in intensity of promotional activities for a given brand increases sales of that brand, while increases in promotional activities of that brand's rivals, in general, decrease brand sales. The exceptions to the latter is that Hunts is not affected by changes in Del Monte promotions, an increase in private label promotional activity increases the demand for Heinz, and the demand for private label ketchup is not affected by the promotional activities of the national brands. Because of its dominant market share in most local markets, retailer promotions of Heinz increases the level of total ketchup expenditures. Thus, the promoting of Heinz by retailers makes the size of the pie bigger for all and as well as changing its distribution. This serves to increase the merchandising elasticities for Heinz, relative to the other brands.

Table 5 presents the parameter estimates of the firm's first-order profit maximization conditions. As discussed earlier, using the estimated values of λ_{ij}, α_j, and mean share values, it is possible to identify the conjectured price elasticity (η_i^c) for each brand as well as the conjectural price reaction elasticities (ϕ_{ji}). If the absolute value of the conjectured price elasticity for brand *i* is greater than one (e.g., firm's operate on the elastic portion of their perceived demand surface), then the value of $\hat{\lambda}_{ii}$ will be positive if $|\hat{\alpha}_i| < |\hat{\eta}_i^c|$, assuming normal goods. This is the case for Hunts and private label ketchup, yielding conjectured price elasticities of –5.51 and –10.72 respectively.[7] However, the value of $\hat{\lambda}_{ii}$ for Heinz is not significantly different than zero, implying that $-\hat{\eta}_i^c = \hat{\alpha}_i$. From the expenditure elasticity formula, $\hat{\alpha}_i$ is equal to $s_i\theta_i$, which must sum to one (Engel Aggregation). Therefore, it is likely that each α_j will be less than one, implying that $|\hat{\eta}_i^c|$ will be less than one when $\hat{\lambda}_{ii}$ is equal to zero. Also, because the value of $\hat{\lambda}_{ii}$ is negative for Del Monte, $|\hat{\alpha}_i| > |\hat{\eta}_i^c|$, implying that $|\hat{\eta}_i^c|$ will again take on a value of less than one. The estimated conjectured price elasticities for Heinz and Del Monte are –0.68 and 4.32 respectively.

It is interesting to note that the estimated values of the own-price demand elasticities for Hunts and private label ketchup in table 3 are much smaller in absolute terms than the estimated conjectured price elasticities. Since all of the cross-price demand elasticities for Hunts and private label ketchup in table 3 are non-negative, and the group expenditure elasticities with respect to Hunts and private label ketchup prices are non-significant (see table 2), implying that the second term in equation (4) is zero, then the conjectured price reaction elasticities must be negative for the conjectured price elasticity to be less than the partial own-price demand

[7] Because equation (43) is quadratic in η_i^c, there exist two different roots. The values of η_i^c reported are the roots that are economically feasible, i.e. greater than –1 in value.

elasticities. Also, from equation (44), if η_i^c > -1 and the brand is normal (i.e., $\hat{a}_i$ > 0), then $\hat{\lambda}_{ij}$ > 0 implies that $\hat{\phi}_{ji}$ < 0. All of the estimated $\hat{\lambda}_{ij}$ are positive for both Hunts and private label ketchup. Thus, both Hunts and private label ketchup have "competitive" reactions towards their rivals price increases. Conversely, if one ignores the fact that the estimated conjectured price elasticities for Heinz and Del Monte are infeasible, then Heinz and Del Monte have different reactions to their rivals. The implied conjectured price reaction elasticities for Heinz are positive (see table 10), implying cooperative behavior. Because the estimated $\hat{\lambda}_{ij}$ coefficients between Del Monte and Heinz and Del Monte and Hunts in table 5 are not significantly different than zero, implying zero conjectured price reaction elasticities, Del Monte does not respond to price changes by Heinz and Hunts. However, the implied conjectured price reaction elasticity between Del Monte and private label ketchup is negative, implying a competitive reaction to changes in private label price.

Focusing on the non-behavior parameters in the firm first-order conditions, most of the estimates have the expected sign (see table 5). Increases in units per volume (U/VOL), the intensity of retailer merchandising (%MER), and the average price reduction of merchandised products (%PRED) all lead to lower brand prices. Increases in the price of tomato paste (TP) lead to higher brand prices, except for having no significant effect on the price of Del Monte. The one variable that had an unexpected sign, was the price of sweeteners (SWT). The brand price of both Hunts and Del Monte decreases as the sweetener prices increases. There was no significant effect of sweetener price on the price of Heinz and private label ketchup. The degree of grocery store concentration had no significant effect on brand prices. Thus, at least for ketchup, retailers in more highly concentrated local markets did not charge relatively higher prices for ketchup. Finally, the marginal cost elasticities for Hunts, Del Monte, and private label ketchup were not significantly different from zero, implying that these brands are produced under (at least locally) constant returns to scale. Interestingly, the estimated marginal cost elasticity for Heinz is positive, suggesting that Heinz is operating in the range of decreasing returns to scale.

5.2 Linear Approximation of Price Reaction Functions

Due of the complexity of the derivation of the non-linear empirical model used above, one can question whether a model that approximates the price reaction elasticities would fit the data just as well. Because the price reaction function can not be explicitly derived using a flexible functional form for consumer demand, an alternative approach would be to take a first-order approximations of the unknown reaction functions (see Cotterill, *et al.*, 1999). Using general notation, the price reaction function for brand/firm j is a function of the prices of all rival brands/firms and cost and demand shifters. Because the Rotterdam demand system is specified as a first-order logarithmic approximation of an unknown demand system, the price reaction functions should also be specified as a first-order logarithmic approximation:

$$d \ln p_j = \sum_{k \neq j}^{n} \lambda_{jk} d \ln p_k + \sum_{z=1}^{t} \nu_{jz} d \ln r_z + \iota_j d \ln X + \sum_{m=1}^{M} \sigma_{jm} d \ln D_m , \tag{60}$$

where p_k, is the price of the kth brand, r_z is the zth input price, X is total ketchup expenditures, and D_m is a vector that includes the demographic variables included in the demand equations and other variables, such as units/volume (U/VOL), percent merchandised (%MER), percent price reduction (%PRED) to control for changes in the price of brand j not associated with changes in rivals' prices. The formal specification of this alternative model is given in table 6.

Tables 7 and 8 give the parameter estimates for this alternative model. Using the first-order logarithmic approximations to the underlying unknown price reaction functions has very little effect on the parameter estimates in the demand and expenditure equations. The compensated and uncompensated price elasticities, and expenditure elasticities given in table 9 are almost identical to those from the original model given in table 3.

The major difference between the two models is the difference between the estimated observed price reaction elasticities and the estimated conjectured price reaction elasticities. Table 10 summarizes the predicted signs of the responses for each of the ketchup brands for the two models. In the conjectures model, Heinz responds positively to changes in all rivals' prices. The same is generally true in the price reaction model, except that Heinz does not respond to a change in the price of Hunts ketchup. For all other brands, the differences between the two models are more noticeable. Hunts is a rival to all other brands in the conjectures model, while the price reaction model predicts that Hunts does not respond to price changes by Heinz and Del Monte and responds cooperatively (positively) to private label price changes. Del Monte is predicted not to respond to price changes by Heinz and Hunts, and to respond competitively to private label price changes in the conjectures model. However, in the price reaction model, Del Monte responds cooperatively with Heinz but does not respond to changes in the price of private label ketchup. Finally, private label ketchup producers are predicted to be a rival to all three national brands of ketchup in the conjectures model. However, in the price reaction model, private label ketchup producers respond cooperatively to price changes by Heinz and Hunts and do not respond to price changes by Del Monte. Thus, each model paints a very different picture about the nature of price coordination between ketchup brands. This result is consistent with Liang who found that estimated conjectured price reaction elasticities are not always consistent with observed price reaction elasticities.

6. Summary and Conclusions

This paper has shown that it is possible to incorporate "flexible" demand systems into empirical models of market power. Using a Rotterdam demand system, a set of first-order conditions for firm profit maximization is derived that are mathematically tractable. However, there are several limitations to this approach. First, the derived first-order conditions are non-linear in the parameters. Without utilizing some simplifying assumptions, these non-linearities make the empirical model difficult to solve. The second limitation, one that is shared with most attempts to estimate firm conduct parameters, is that the estimated conjectural price elasticity and price reaction elasticities are treated as constant. It is likely that these elasticities, and therefore firm conduct, will vary over time and potentially across local markets as well. One reason why firm conduct is treated as a constant over time and local markets is that the number of unknown firm conduct parameters likely exceeds the number of observations in the data.

For this research on estimating conjectural elasticities to really bear fruit, more work is needed estimating conduct parameters that vary across time and markets and to incorporate all available information in the estimation process (e.g., that parameter λ_{jj} is really a non-linear function of α_j and η_j, which is itself a function of the estimated price and expenditure elasticities, and the firm's conjectural elasticities.) One promising avenue to deal with these issues may be to utilize a maximum entropy approach (see Golan, *et al.*). One of the strengths of maximum entropy is the case of ill-posed problems, such as when the number of parameters to be estimated exceeds the number of observations in the data set. In addition, because it is implemented as a mathematical program, it may be possible to include all λ_{ii} model parameter definitions as model constraints.

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Table 1. Ketchup Brand Pricing Model

Ketchup Demand Equations

$$\begin{aligned}\bar{s}_{HZ}\, d\ln q_{HZ} = {} & \alpha_1\left(d\ln X - \bar{s}_{HZ}\, d\ln p_{HZ} - \bar{s}_{HU}\, d\ln p_{HU} - \bar{s}_{DM}\, d\ln p_{DM} - \bar{s}_{PL}\, d\ln p_{PL}\right) + \\ & \delta_{11}\left(d\ln p_{HZ} - d\ln p_{PL}\right) + \delta_{12}\left(d\ln p_{HU} - d\ln p_{PL}\right) + \delta_{13}\left(d\ln p_{DM} - d\ln p_{PL}\right) + \\ & \omega_1 d\ln PHSP + \kappa_1 d\ln AGE + \tau_{11} d\ln \%MER_{HZ} + \tau_{12} d\ln \%MER_{HU} + \\ & \tau_{13} d\ln \%MER_{DM} + \tau_{14} d\ln \%MER_{PL} + e_1\end{aligned}$$

$$\begin{aligned}\bar{s}_{HU}\, d\ln q_{HU} = {} & \alpha_2\left(d\ln X - \bar{s}_{HZ}\, d\ln p_{HZ} - \bar{s}_{HU}\, d\ln p_{HU} - \bar{s}_{DM}\, d\ln p_{DM} - \bar{s}_{PL}\, d\ln p_{PL}\right) + \\ & \delta_{21}\left(d\ln p_{HZ} - d\ln p_{PL}\right) + \delta_{22}\left(d\ln p_{HU} - d\ln p_{PL}\right) + \delta_{23}\left(d\ln p_{DM} - d\ln p_{PL}\right) + \\ & \omega_2 d\ln PHSP + \kappa_2 d\ln AGE + \tau_{21} d\ln \%MER_{HZ} + \tau_{22} d\ln \%MER_{HU} + \\ & \tau_{23} d\ln \%MER_{DM} + \tau_{24} d\ln \%MER_{PL} + e_2\end{aligned}$$

$$\begin{aligned}\bar{s}_{DM}\, d\ln q_{DM} = {} & \alpha_3\left(d\ln X - \bar{s}_{HZ}\, d\ln p_{HZ} - \bar{s}_{HU}\, d\ln p_{HU} - \bar{s}_{DM}\, d\ln p_{DM} - \bar{s}_{PL}\, d\ln p_{PL}\right) + \\ & \delta_{31}\left(d\ln p_{HZ} - d\ln p_{PL}\right) + \delta_{32}\left(d\ln p_{HU} - d\ln p_{PL}\right) + \delta_{33}\left(d\ln p_{DM} - d\ln p_{PL}\right) + \\ & \omega_3 d\ln PHSP + \kappa_3 d\ln AGE + \tau_{31} d\ln \%MER_{HZ} + \tau_{32} d\ln \%MER_{HU} + \\ & + \tau_{33} d\ln \%MER_{DM} + \tau_{34} d\ln \%MER_{PL} + e_3\end{aligned}$$

Manufacturer Profit Maximization First-Order Conditions

$$\begin{aligned}d\ln p_{HZ} = {} & \frac{1}{(1-\lambda_{11})s_{PL} + s_{HZ}\lambda_{14}}\Big\{s_{PL}\left(\nu_{11} d\ln TP + \nu_{12} d\ln SWT\right) + \\ & \left[s_{PL}\left(\eta^{HZ}_{mc} + \lambda_{11}\right) - s_{HZ}\lambda_{14}\right] d\ln q_{HZ} + \left(s_{PL}\lambda_{12} - s_{HU}\lambda_{14}\right)\left(d\ln p_{HU} + d\ln q_{HU}\right) + \\ & \left(s_{PL}\lambda_{13} - s_{DM}\lambda_{14}\right)\left(d\ln p_{DM} + d\ln q_{DM}\right) + \left[\lambda_{14} - s_{PL}\left(\lambda_{11} + \lambda_{12} + \lambda_{13} + \lambda_{14}\right)\right] d\ln x\Big\} + \\ & \psi_1 d\ln U/VOL_{HZ} + \chi_1 d\ln PRED_{HZ} + \vartheta_1 d\ln \%MER_{HZ} + \xi_1 d\ln CR4 + e_4\end{aligned}$$

$$\begin{aligned}d\ln p_{HU} = {} & \frac{1}{(1-\lambda_{22})s_{PL} + s_{HU}\lambda_{24}}\Big\{s_{PL}\left(\nu_{21} d\ln TP + \nu_{22} d\ln SWT\right) + \\ & \left[s_{PL}\left(\eta^{HU}_{mc} + \lambda_{22}\right) - s_{HU}\lambda_{24}\right] d\ln q_{HU} + \left(s_{PL}\lambda_{21} - s_{HZ}\lambda_{24}\right)\left(d\ln p_{HZ} + d\ln q_{HZ}\right) + \\ & \left(s_{PL}\lambda_{23} - s_{DM}\lambda_{24}\right)\left(d\ln p_{DM} + d\ln q_{DM}\right) + \left[\lambda_{24} - s_{PL}\left(\lambda_{21} + \lambda_{22} + \lambda_{23} + \lambda_{24}\right)\right] d\ln x\Big\} + \\ & \psi_2 d\ln U/VOL_{HU} + \chi_2 d\ln PRED_{HU} + \vartheta_2 d\ln \%MER_{HU} + \xi_2 d\ln CR4 + e_5\end{aligned}$$

$$\begin{aligned}d\ln p_{DM} = {} & \frac{1}{(1-\lambda_{33})s_{PL} + s_{DM}\lambda_{34}}\Big\{s_{PL}\left(\nu_{31} d\ln TP + \nu_{32} d\ln SWT\right) + \\ & \left[s_{PL}\left(\eta^{DM}_{mc} + \lambda_{33}\right) - s_{DM}\lambda_{34}\right] d\ln q_{DM} + \left(s_{PL}\lambda_{31} - s_{HZ}\lambda_{34}\right)\left(d\ln p_{HZ} + d\ln q_{HZ}\right) + \\ & \left(s_{PL}\lambda_{32} - s_{HU}\lambda_{34}\right)\left(d\ln p_{HU} + d\ln q_{HU}\right) + \left[\lambda_{34} - s_{PL}\left(\lambda_{31} + \lambda_{32} + \lambda_{33} + \lambda_{34}\right)\right] d\ln x\Big\} + \\ & \psi_3 d\ln U/VOL_{DM} + \chi_3 d\ln PRED_{DM} + \vartheta_3 d\ln \%MER_{DM} + \xi_3 d\ln CR4 + e_6\end{aligned}$$

$$\begin{aligned}d\ln p_{PL} = {} & \frac{1}{\left(1+\eta^{PL}_{mc}\right)}\left(\nu_{41} d\ln TP + \nu_{42} d\ln SWT\right) + \frac{1}{\left(1+\eta^{PL}_{mc}\right)s_{PL}}\Big\{\left(s_{PL}\lambda_{41} - s_{HZ}\left(\eta^{PL}_{mc} + \lambda_{44}\right)\right) \\ & \left(d\ln p_{HZ} + d\ln q_{HZ}\right) + \left(s_{PL}\lambda_{42} - s_{HU}\left(\eta^{PL}_{mc} + \lambda_{44}\right)\right)\left(d\ln p_{HU} + d\ln q_{HU}\right) + \\ & \left(s_{PL}\lambda_{43} - s_{DM}\left(\eta^{PL}_{mc} + \lambda_{44}\right)\right)\left(d\ln p_{DM} + d\ln q_{DM}\right) + \\ & \left[\eta^{PL}_{mc} + \lambda_{44} - s_{PL}\left(\lambda_{41} + \lambda_{42} + \lambda_{43} + \lambda_{44}\right)\right] d\ln x\Big\} + \\ & \psi_4 d\ln U/VOL_{PL} + \chi_4 d\ln PRED_{PL} + \vartheta_4 d\ln \%MER_{PL} + \xi_4 d\ln CR4 + e_7\end{aligned}$$

Ketchup Expenditure Equation

$$\begin{aligned}d\ln X = {} & \mu_1 d\ln p_{HZ} + \mu_2 d\ln p_{HU} + \mu_3 d\ln p_{DM} + \mu_4 d\ln p_{PL} + \beta_1 d\ln p^s_1 + \beta_2 d\ln p^s_2 + \\ & \beta_3 d\ln p^s_3 + \beta_4 d\ln p^s_4 + \beta_5 d\ln p^s_5 + \rho_1 d\ln INC + \rho_2 d\ln IU10K + \rho_3 d\ln IO50K + \\ & \gamma_1 d\ln PHSP + \gamma_2 d\ln AGE + \gamma_3 d\ln \%MER_{HZ} + \gamma_4 d\ln \%MER_{HU} + \gamma_5 d\ln \%MER_{DM} + \\ & \gamma_6 d\ln \%MER_{PL} + e_8\end{aligned}$$

(Continues)

Table 1 Continued

Variable Definitions:	
HZ	Heinz
HU	Hunts
DM	Del Monte
PL	Private Label
s_j	market share of *j*th brand
q_j	quantity of *j*th brand
X	total ketchup expenditures
p_j	price of *j*th brand
PHSP	percentage of regional population that is Hispanic
AGE	median age in region
TP	price of tomato paste
SWT	price of sweetener
U/VOL	units per volume
PRED	average price reduction of products merchandised
%MER	percentage of sales with retailer merchandising
CR4	four firm concentration ratio of local grocery stores
p_k^s	average price of *k*th substitute or complement for ketchup
INC	median household income
IU10K	percent of households with income under $10,000
IO50K	percent of households with income over $50,000

Table 2. Demand and Expenditure Equation Parameter Estimates

	Dependent Variables[a]			
	Quantity			
Independent Variables[a]	Heinz	Hunts	Del Monte	Expenditure
Brand Prices				
Heinz	-0.43			-0.43
	(0.032)*			(0.12)*
Hunts	0.18	-0.25		0.063
	(0.026)*	(0.029)*		(0.092)
Del Monte	0.10	0.042	-0.18	0.033
	(0.016)*	(0.015)*	(0.016)*	(0.077)
Private Label				-0.21
				(0.16)
Real Expenditure[b]	0.68	0.14	0.010	
	(0.019)*	(0.018)*	(0.011)*	
Percent Hispanic Population	-0.076	0.059	0.013	0.021
	(0.050)	(0.051)	(0.030)	(0.15)
Median Age of Household Head	-0.24	0.10	0.0005	0.80
	(0.28)	(0.29)	(0.17)	(0.82)
Other Group Prices[c]				
Mayonnaise				-0.35
				(0.11)*
Mustard				-0.38
				(0.082)*
Barbecue Sauce				0.35
				(0.065)*
Hot Sauce				-0.044
				(0.067)
Steak Sauce				1.52
				(0.11)*
Median Household Income				-0.20
				(0.092)**
% Households < $10,000				0.27
				(0.22)
% Households > $50,000				-0.090
				(0.18)
Percent Sales Merchandised[d]				
Heinz	0.015	-0.012	-0.0077	0.050
	(0.0049)*	(0.0045)*	(0.0028)*	(0.013)*
Hunts	-0.025	0.037	-0.0043	0.014
	(0.0045)*	(0.0047)*	(0.0026)***	(0.013)
Del Monte	-0.0093	-0.00013	0.013	0.0049
	(0.0038)**	(0.0037)	(0.0029)*	(0.013)
Private Label	-0.0033	0.0021	0.00038	-0.0045
	(0.0024)	(0.0023)	(0.0014)	(0.0095)
Equation Adjusted R^2	.901	.596	.509	.423

* Significant at 1% level
** Significant at 5% level
a All variables are measured in logarithmic differences.
b Logarithmic change in real ketchup expenditures.
c Aggregate price of complementary or substitute commodity groups.

Table 3. Estimated Brand Price and Expenditure Demand Elasticities

	Price				Expenditure
	Heinz	Hunts	Del Monte	Private Label	
		Expenditure Constant[a]			
Heinz	-1.43	0.058	0.072	0.12	1.19
	(0.057)*	(0.046)	(0.029)*	(0.028)*	(0.033)*
Hunts	0.46	-1.32	0.14	0.069	0.65
	(0.12)*	(0.14)*	(0.069)**	(0.054)	(0.082)*
Del Monte	0.48	0.22	-2.07	0.22	1.15
	(0.18)*	(0.17)	(0.18)*	(0.10)**	(0.13)*
Private Label	0.87	0.13	0.21	-1.84	0.63
	(0.13)*	(0.094)	(0.076)*	(0.13)*	(0.072)*
		Uncompensated[b]			Income[c]
Heinz	-1.94	0.13	0.11	-0.14	-0.23
	(0.16)*	(0.12)	(0.095)	(0.18)	(0.11)**
Hunts	0.18	-1.27	0.16	-0.071	-0.13
	(0.16)	(0.15)*	(0.086)***	(0.12)	(0.062)**
Del Monte	-0.016	0.29	-2.03	-0.027	-0.23
	(0.24)	(0.19)	(0.20)*	(0.21)	(0.11)**
Private Label	0.60	0.17	0.23	-1.98	-0.13
	(0.16)*	(0.12)	(0.095)**	(0.18)*	(0.060)**

* Significant at 1% level

** Significant at 5% level

*** Significant at 10% level

[a] Price elasticities are computed holding total ketchup expenditures constant.

[b] Price elasticities taken into account the estimated effect of brand price changes on total ketchup expenditures. The uncompensated price elasticities are computed as the compensated (or expenditure constant) price elasticity plus the expenditure elasticity times the group expenditure elasticity with respect to a brand price change. In other words:

$$e_{ij}^{u} = e_{ij} + q_i m_j \ ,$$

where m_j is coefficient on brand price *j* in the expenditure equation.

[c] The income elasticity is computed as the expenditure elasticity times the elasticity of ketchup expenditures with respect to median income (i.e., the coefficient on median income in the expenditure equation).

Table 4. Estimated Brand Merchandising Elasticities

	Heinz	Hunts	Del Monte	Private Label
		Expenditure Constant[a]		
Heinz	0.026	-0.044	-0.016	-0.0058
	(0.085)*	(0.0078)*	(0.0067)**	(0.0042)
Hunts	-0.054	0.17	-0.0006	0.0099
	(0.021)*	(0.022)*	(0.017)	(0.011)
Del Monte	-0.086	-0.048	0.14	0.0043
	(0.031)*	(0.029)***	(0.033)*	(0.016)
Private Label	0.039	-0.060	-0.027	0.0066
	(0.018)**	(0.014)*	(0.014)**	(0.0087)
		Total Effects[b]		
Heinz	0.085	-0.027	-0.010	-0.011
	(0.017)*	(0.017)***	(0.017)	(0.012)
Hunts	-0.021	0.18	0.0026	-0.007
	(0.021)	(0.023)*	(0.019)	(0.012)
Del Monte	-0.028	-0.031	0.15	-0.0009
	(0.031)	(0.032)	(0.035)*	(0.019)
Private Label	0.071	-0.051	-0.024	0.0037
	(0.019)*	(0.017)*	(0.017)	(0.011)

* Significant at 1% level

** Significant at 5% level

*** Significant at 10% level

[a] Elasticities are computed holding total ketchup expenditures constant (t_{ij}/s_i)

[b] Total elasticities take into account the effect of merchandising on total ketchup expenditures: $(t_{ij} + a_i g_j)/s_i$. (See table 1 for definitions.)

Table 5. Parameter Estimates for Profit Maximization First-Order Conditions

Independent Variables or Parameters	Price Dependent Variables			
	Heinz	Hunts	Del Monte	Private Label
λ_{ij}				
Heinz	-0.048	0.77	-0.27	0.30
	(0.15)	(0.26)*	(0.27)	(0.14)**
Hunts	0.17	0.22	-0.065	0.11
	(0.058)*	(0.096)**	(0.10)	(0.054)**
Del Monte	0.057	0.092	-0.19	0.031
	(0.023)**	(0.029)*	(0.065)*	(0.017)***
Private Label	0.39	0.34	0.10	0.11
	(0.055)*	(0.072)*	(0.061)	(0.040)*
Marginal Cost Elasticity (η_{mc})	0.13	-0.043	-0.031	0[a]
	(0.041)*	(0.033)	(0.045)	
Input Prices				
Tomato Paste	0.11	0.16	0.073	0.094
	(0.049)**	(0.067)**	(0.073)	(0.043)**
Sweeteners	0.0049	-0.034	-0.087	-0.0021
	(0.013)	(0.015)**	(0.019)*	(0.011)
Units/Volume	-0.094	-0.44	-0.36	-0.59
	(0.028)*	(0.071)*	(0.061)*	(0.059)*
Percent Merchandised	-0.017	-0.035	0.0004	-0.029
	(0.0036)*	(0.0092)*	(0.014)	(0.0033)*
Percent Price Reduction	-0.023	-0.063	-0.042	-0.038
	(0.0032)*	(0.0092)*	(0.0073)*	(0.0042)*
Grocery Store CR4	0.0011	-0.019	0.053	0.016
	(0.025)	(0.074)	(0.087)	(0.067)
Equation Adjusted R^2	0.878	0.620	0.593	0.359

* Significant at 1% level

** Significant at 5% level

*** Significant at 10% level

[a] Initial attempts to estimate the marginal cost elasticity for private label products lead to unstable parameters estimates for the entire equation. Because the initial estimates indicated the marginal cost elasticity for private label ketchup not to be significantly different than zero, (e.g., private label ketchup is produced at a point of constant returns to scale technology) this parameter is set equal to zero and the model re-estimated. This choice had little or no effects on parameter estimates in other equations.

Table 6. Ketchup Brand Pricing Model with Approximated Price Reaction Functions

Ketchup Demand Equations

$$\bar{s}_{HZ}\, d\ln q_{HZ} = \mathrm{a}_1 (d\ln X - \bar{s}_{HZ}\, d\ln p_{HZ} - \bar{s}_{HU}\, d\ln p_{HU} - \bar{s}_{DM}\, d\ln p_{DM} - \bar{s}_{PL}\, d\ln p_{PL}) +$$
$$\mathrm{d}_{11}(d\ln p_{HZ} - d\ln p_{PL}) + \mathrm{d}_{12}(d\ln p_{HU} - d\ln p_{PL}) + \mathrm{d}_{13}(d\ln p_{DM} - d\ln p_{PL}) +$$
$$\mathrm{w}_1 d\ln PHSP + \mathrm{k}_1 d\ln AGE + \mathrm{t}_{11} d\ln \%MER_{HZ} + \mathrm{t}_{12} d\ln \%MER_{HU} +$$
$$\mathrm{t}_{13} d\ln \%MER_{DM} + \mathrm{t}_{14} d\ln \%MER_{PL} + e_1$$

$$\bar{s}_{HU}\, d\ln q_{HU} = \mathrm{a}_2 (d\ln X - \bar{s}_{HZ}\, d\ln p_{HZ} - \bar{s}_{HU}\, d\ln p_{HU} - \bar{s}_{DM}\, d\ln p_{DM} - \bar{s}_{PL}\, d\ln p_{PL}) +$$
$$\mathrm{d}_{21}(d\ln p_{HZ} - d\ln p_{PL}) + \mathrm{d}_{22}(d\ln p_{HU} - d\ln p_{PL}) + \mathrm{d}_{23}(d\ln p_{DM} - d\ln p_{PL}) +$$
$$\mathrm{w}_2 d\ln PHSP + \mathrm{k}_2 d\ln AGE + \mathrm{t}_{21} d\ln \%MER_{HZ} + \mathrm{t}_{22} d\ln \%MER_{HU} +$$
$$\mathrm{t}_{23} d\ln \%MER_{DM} + \mathrm{t}_{24} d\ln \%MER_{PL} + e_2$$

$$\bar{s}_{DM}\, d\ln q_{DM} = \mathrm{a}_3 (d\ln X - \bar{s}_{HZ}\, d\ln p_{HZ} - \bar{s}_{HU}\, d\ln p_{HU} - \bar{s}_{DM}\, d\ln p_{DM} - \bar{s}_{PL}\, d\ln p_{PL}) +$$
$$\mathrm{d}_{31}(d\ln p_{HZ} - d\ln p_{PL}) + \mathrm{d}_{32}(d\ln p_{HU} - d\ln p_{PL}) + \mathrm{d}_{33}(d\ln p_{DM} - d\ln p_{PL}) +$$
$$\mathrm{w}_3 d\ln PHSP + \mathrm{k}_3 d\ln AGE + \mathrm{t}_{31} d\ln \%MER_{HZ} + \mathrm{t}_{32} d\ln \%MER_{HU} +$$
$$+\mathrm{t}_{33} d\ln \%MER_{DM} + \mathrm{t}_{34} d\ln \%MER_{PL} + e_3$$

First-Order Approximations of Manufacturer Price Reaction Functions

$$d\ln p_{HZ} = \mathrm{l}_{12} d\ln p_{HU} + \mathrm{l}_{13} d\ln p_{DM} + \mathrm{l}_{14} d\ln p_{PL} + \mathrm{n}_{11} d\ln TP + \mathrm{n}_{12} d\ln SWT + \mathrm{i}_{HZ} d\ln X +$$
$$\mathrm{y}_1 d\ln U/VOL_{HZ} + \mathrm{c}_1 d\ln PRED_{HZ} + \mathrm{J}_1 d\ln \%MER_{HZ} + \mathrm{x}_1 d\ln CR4 + \mathrm{p}_{HZ} d\ln PHSP +$$
$$\mathrm{s}_{HZ} d\ln AGE + e_4$$

$$d\ln p_{HU} = \mathrm{l}_{21} d\ln p_{HZ} + \mathrm{l}_{23} d\ln p_{DM} + \mathrm{l}_{24} d\ln p_{PL} + \mathrm{n}_{21} d\ln TP + \mathrm{n}_{22} d\ln SWT + \mathrm{i}_{HU} d\ln X +$$
$$\mathrm{y}_2 d\ln U/VOL_{HU} + \mathrm{c}_2 d\ln PRED_{HU} + \mathrm{J}_2 d\ln \%MER_{HU} + \mathrm{x}_2 d\ln CR4 + \mathrm{p}_{HU} d\ln PHSP +$$
$$\mathrm{s}_{HU} d\ln AGE + e_4$$

$$d\ln p_{DM} = \mathrm{l}_{31} d\ln p_{HZ} + \mathrm{l}_{32} d\ln p_{HU} + \mathrm{l}_{34} d\ln p_{PL} + \mathrm{n}_{31} d\ln TP + \mathrm{n}_{32} d\ln SWT + \mathrm{i}_{DM} d\ln X +$$
$$\mathrm{y}_3 d\ln U/VOL_{DM} + \mathrm{c}_3 d\ln PRED_{DM} + \mathrm{J}_3 d\ln \%MER_{DM} + \mathrm{x}_3 d\ln CR4 + \mathrm{p}_{DM} d\ln PHSP +$$
$$\mathrm{s}_{DM} d\ln AGE + e_6$$

$$d\ln p_{PL} = \mathrm{l}_{41} d\ln p_{HZ} + \mathrm{l}_{42} d\ln p_{HU} + \mathrm{l}_{43} d\ln p_{DM} + \mathrm{n}_{41} d\ln TP + \mathrm{n}_{42} d\ln SWT + \mathrm{i}_{PL} d\ln X +$$
$$\mathrm{y}_4 d\ln U/VOL_{PL} + \mathrm{c}_4 d\ln PRED_{PL} + \mathrm{J}_4 d\ln \%MER_{PL} + \mathrm{x}_4 d\ln CR4 + \mathrm{p}_{PL} d\ln PHSP +$$
$$\mathrm{s}_{PL} d\ln AGE + e_7$$

Ketchup Expenditure Equation

$$d\ln X = \mathrm{m}_1 d\ln p_{HZ} + \mathrm{m}_2 d\ln p_{HU} + \mathrm{m}_3 d\ln p_{DM} + \mathrm{m}_4 d\ln p_{PL} + \mathrm{b}_1 d\ln p_1^s + \mathrm{b}_2 d\ln p_2^s +$$
$$\mathrm{b}_3 d\ln p_3^s + \mathrm{b}_4 d\ln p_4^s + \mathrm{b}_5 d\ln p_5^s + \mathrm{r}_1 d\ln INC + \mathrm{r}_2 d\ln IU10K + \mathrm{r}_3 d\ln IO50K +$$
$$\mathrm{g}_1 d\ln PHSP + \mathrm{g}_2 d\ln AGE + \mathrm{g}_3 d\ln \%MER_{HZ} + \mathrm{g}_4 d\ln \%MER_{HU} + \mathrm{g}_5 d\ln \%MER_{DM} +$$
$$\mathrm{g}_6 d\ln \%MER_{PL} + e_8$$

(continues)

Table 6. (continued)

Variable Definitions:	
HZ	Heinz
HU	Hunts
DM	Del Monte
PL	Private Label
s_j	market share of *j*th brand
q_j	quantity of *j*th brand
X	total ketchup expenditures
p_j	price of *j*th brand
PHSP	percentage of regional population that is Hispanic
AGE	median age in region
TP	price of tomato paste
SWT	price of sweetener
U/VOL	units per volume
PRED	average price reduction of products merchandised
%MER	percentage of sales with merchandising
CR4	four firm concentration ratio of local grocery stores
p_k^s	average price of *k*th substitute or complement for ketchup
INC	median household income
IU10K	percent of households with income under $10,000
IO50K	percent of households with income over $50,000

Table 7. Demand and Expenditure Equation Parameter Estimates for Brand Pricing Model with Approximated Price Reaction Functions

	Dependent Variables[a]			
	Quantity			
Independent Variables[a]	Heinz	Hunts	Del Monte	Expenditure
Brand Prices				
Heinz	-0.40	--[b]	--[b]	-0.45
	(0.035)*			(0.12)*
Hunts	0.16	-0.24	--[b]	0.036
	(0.027)*	(0.030)*		(0.091)
Del Monte	0.10	0.043	-0.18	0.031
	(0.018)*	(0.016)*	(0.017)*	(0.078)
Private Label				-0.18
				(0.15)
Real Expenditure[c]	0.68	0.12	0.010	
	(0.019)*	(0.018)*	(0.012)*	
Percent Hispanic Population	-0.10	0.068	0.012	0.017
	(0.055)***	(0.053)	(0.034)	(0.15)
Median Age of Household Head	-0.55	0.14	0.17	0.76
	(0.31)***	(0.30)	(0.19)	(0.83)
Other Group Prices[d]				
Mayonnaise				-0.29
				(0.10)*
Mustard				-0.35
				(0.081)*
Barbecue Sauce				0.44
				(0.064)*
Hot Sauce				-0.013
				(0.066)
Steak Sauce				1.50
				(0.11)*
Median Household Income				-0.20
				(0.090)**
% Households < $10,000				0.20
				(0.22)
% Households > $50,000				-0.010
				(0.18)
Percent Sales Merchandised				
Heinz	0.016	-0.0063	-0.0071	0.062
	(0.0050)*	(0.0045)	(0.0030)**	(0.013)*
Hunts	-0.025	0.038	-0.0034	0.017
	(0.0048)*	(0.0048)*	(0.0028)	(0.013)
Del Monte	-0.0093	0.00078	0.013	0.0095
	(0.0041)**	(0.0037)	(0.0030)*	(0.013)
Private Label	-0.0075	0.0018	-0.0033	-0.0045
	(0.0025)*	(0.0023)	(0.0016)**	(0.0092)
Equation Adjusted R^2	.901	.583	.514	.428

* Significant at 1% level
** Significant at 5% level
[a] All variables are measured in logarithmic differences.
[b] Parameter estimates are determined via symmetry conditions.
[c] Logarithmic change in real ketchup expenditures.
[d] Aggregate price of complementary or substitute commodity groups.

Table 8. Parameter Estimates for Profit Maximization First-Order Conditions for Brand Pricing Model with Approximated Price Reaction Functions

	Price Dependent Variables			
Independent Variables or Parameters	Heinz	Hunts	Del Monte	Private Label
λ_{ij}				
Heinz		0.056	0.23	0.094
		(0.045)	(0.067)*	(0.036)*
Hunts	0.024		0.031	0.049
	(0.031)		(0.049)	(0.026)***
Del Monte	0.062	0.031		0.011
	(0.024)*	(0.026)		(0.002)
Private Label	0.14	0.11	-0.073	
	(0.048)*	(0.052)**	(0.076)	
Input Prices				
Tomato Paste	0.020	0.11	0.038	0.091
	(0.049)	(0.051)**	(0.076)	(0.041)**
Sweeteners	0.025	-0.030	-0.053	0.0051
	(0.013)***	(0.012)**	(0.018)*	(0.010)
Units/Volume	-0.40	-0.53	-0.66	-0.64
	(0.079)*	(0.044)*	(0.061)*	(0.057)*
Percent Merchandised	-0.067	-0.072	-0.093	-0.032
	(0.0057)*	(0.0052)*	(0.0068)*	(0.0028)*
Percent Price Reduction	-0.088	-0.10	-0.086	-0.042
	(0.0054)*	(0.0058)*	(0.0067)*	(0.0042)*
Grocery Store CR4	0.026	0.027	0.11	0.048
	(0.071)	(0.075)	(0.11)	(0.061)
Ketchup Expenditures	0.038	0.0060	0.14	0.056
	(0.031)	(0.027)	(0.040)*	(0.022)*
Percent Hispanic Population	0.062	0.079	-0.053	0.078
	(0.066)	(0.074)	(0.11)	(0.057)
Median Age of Household Head	0.43	1.16	0.38	0.076
	(0.38)	(0.43)*	(0.62)	(0.33)
Equation Adjusted R^2	0.529	0.607	0.471	0.474

* Significant at 1% level
** Significant at 5% level
*** Significant at 10% level

Table 9. Estimated Brand Price and Expenditure Demand Elasticities for Model with Approximated Price Reaction Functions

	Price				Expenditure
	Heinz	Hunts	Del Monte	Private Label	
	Expenditure Constant[a]				
Heinz	-1.39	0.024	0.063	0.11	1.19
	(0.060)*	(0.048)	(0.032)**	(0.032)*	(0.033)*
Hunts	0.43	-1.24	0.15	0.12	0.54
	(0.13)*	(0.14)*	(0.073)**	(0.067)***	(0.083)*
Del Monte	0.44	0.23	-2.15	0.36	1.12
	(0.20)**	(0.18)	(0.19)*	(0.13)*	(0.13)*
Private Label	0.72	0.15	0.29	-1.99	0.84
	(0.14)*	(0.12)	(0.092)*	(0.16)*	(0.072)*
	Uncompensated[b]				Income[c]
Heinz	-1.91	0.067	0.10	-0.10	-0.24
	(0.16)*	(0.12)	(0.097)	(0.19)	(0.11)**
Hunts	0.19	-1.22	0.16	0.022	-0.11
	(0.16)	(0.15)*	(0.087)***	(0.11)	(0.052)**
Del Monte	-0.059	0.27	-2.12	0.16	-0.22
	(0.26)	(0.20)	(0.20)*	(0.22)	(0.10)**
Private Label	0.35	0.18	0.32	-2.14	-0.16
	(0.18)**	(0.14)	(0.11)*	(0.21)*	(0.077)**

* Significant at 1% level
** Significant at 5% level
*** Significant at 10% level
[a] Price elasticities are computed holding total ketchup expenditures constant.
[b] Price elasticities taken into account the estimated effect of brand price changes on total ketchup expenditures. The uncompensated price elasticities are computed as the compensated (or expenditure constant) price elasticity plus the expenditure elasticity times the group expenditure elasticity with respect to a brand price change. In other words:

$$e^{u}_{ij} = e_{ij} + q_i m_j \ ,$$

where m_j is coefficient on brand price *j* in the expenditure equation.
[c] The income elasticity is computed as the expenditure elasticity times the elasticity of ketchup expenditures with respect to median income (i.e., the coefficient on median income in the expenditure equation).

Table 10. Predicted Price Responses Between Brands[a]

	Heinz	Hunts	Del Monte	Private Label
	Model with Derived Firm FOC			
Heinz		(+)	(+)	(+)
Hunts	(-)		(-)	(-)
Del Monte	0	0		(-)
Private Label	(-)	(-)	(-)	
	Model with Approximated Price Reaction Functions			
Heinz		0	(+)	(+)
Hunts	0		0	(+)
Del Monte	(+)	0		0
Private Label	(+)	(+)	0	

[a] This table presents the predicted direction of price responsiveness to a change in a rival's price. A (+) indicates that the firm follows the rivals price change, (-) indicates that the firm takes the opposite action of a rival, and 0 indicates that the firm does not respond to a rival's price change.

Appendix

Table A1. Regional Markets Included in Study

Region
Atlanta
Baltimore/Washington D.C.
Birmingham
Chicago
Cincinnati/Dayton
Columbus, OH
Dallas/Ft. Worth
Denver
Detroit
Grand Rapids
Hartford/Springfield
Houston
Indianapolis
Jacksonville
Kansas City
Little Rock
Los Angeles
Louisville
Memphis
Miami/Ft. Lauderdale
Milwaukee
Minneapolis/St. Paul
Nashville
New Orleans/Mobile
New York
Oklahoma City
Omaha
Orlando
Philadelphia
Phoenix/Tucson
Portland, OR
Raleigh/Greensboro
Sacramento
Salt Lake City
San Antonio
San Diego
San Francisco/Oakland
Seattle/Tacoma
St. Louis
Tampa/St. Petersburg
Wichita

Table A2. Descriptive Statistics

Name	Mean	Standard Deviation	Minimum	Maximum
Budget Shares (s_j)				
Heinz	0.572	0.121	0.288	0.910
Hunts	0.217	0.109	0.036	0.515
Del Monte	0.090	0.065	0.003	0.307
Private Label	0.121	0.049	0.014	0.292
Ketchup Brand Prices (p_j)		$/lb.		
Heinz	0.797	0.080	0.564	1.073
Hunts	0.712	0.102	0.478	1.022
Del Monte	0.631	0.097	0.427	0.937
Private Label	0.564	0.079	0.412	0.842
Units per Volume (*U/VOL*)				
Heinz	1.89	0.08	1.64	2.13
Hunts	1.87	0.16	1.48	2.31
Del Monte	1.87	0.13	1.07	2.35
Private Label	1.80	0.10	1.51	2.22
Percent of Volume Merchandised (*%MER*)				
Heinz	38.3	14.7	0.5	86.6
Hunts	47.7	16.4	5.7	89.0
Del Monte	56.6	17.7	3.8	98.3
Private Label	35.4	16.7	2.2	92.3
Average Price Reduction of Products Merchandised (*PRED*)				
Heinz	21.4	7.5	7.4	49.8
Hunts	21.3	7.5	6.6	45.1
Del Monte	22.0	7.7	6.7	52.5
Private Label	19.0	6.6	6.0	46.1
Total Ketchup Expenditures (*X*)	1232730.4	1183249.9	181208.1	8050906.2
Hispanic Percent of Population (*HISP*)	7.8	10.0	0.1	48.7
Median age (*AGE*)	33.0	2.4	24.1	41.8
Median Household Income (*INC*)	32359.9	7047.7	20729.0	53429.0
Percent of Households with Income < $10,000 (*IU10K*)	15.0	3.2	7.9	24.2
Percent of Households with Income > $50,000 (*IU10K*)	24.3	6.4	12.4	44.9
Price of Tomato Paste (*TP*)	0.423	0.098	0.278	0.58
Price of Sweetener (*SWT*)	21.7	3.0	14.4	27.0
Local Grocery 4-Firm Concentration (*CR4*)	64.9	12.5	30.2	88.1

Average Price of Substitutes (p_s^k)		$/lb.		
Mayonnaise	0.969	0.123	0.639	1.272
Mustard	1.222	0.219	0.736	1.838
Barbecue Sauce	1.201	0.173	0.844	1.794
Hot Sauce	2.063	0.658	1.145	5.155
Steak/Worcestershire Sauce	3.500	0.415	2.332	4.611

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