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Optimal Brand and Generic Advertising Policies in a Dynamic Differentiated Product Oligopoly

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Abstract

In some product categories, generic advertising is used to increase market demand of the category and at the same time brand advertising is used to entice consumers to choose the advertised brand over competing brands. This paper empirically investigates the optimal levels of brand and generic advertising in a dynamic differentiated product oligopoly. A nested logit demand system incorporating brand and generic advertising goodwill stocks is specified and estimated without imposing any supply-side restrictions. Demand side parameters are then used to calibrate a dynamic game of brand and generic advertising that takes into account the vertical relationship between manufacturers and retailers. Estimates from the fluid milk product category indicate that brand advertising is effective for increasing brand level demand and generic advertising has a differential effect on individual brands. On the supply side, we found that it is not optimal for brand manufacturers to advertise in the presence of generic advertising.

Key words: brand advertising; generic advertising; dynamic oligopoly; Markov perfect equilibrium; Bayesian analysis

1. Introduction

In some product categories such as fluid milk, orange juice, and beef, one observes hundreds of millions of dollars of generic and brand advertising. The primary goal of generic advertising is to increase market demand of a product without necessarily influencing the market share for any one brand. In contrast, brand advertising provides consumers with information about a brand that differentiates it from its competitors', thereby persuading them to choose the advertised brand over competing brands. When both types of advertising exist in a market, there are concerns that generic advertising might affect the level of brand advertising (Chakravart and Janiszewski 2004). Whether this is true or not depends mainly on how generic and brand advertising affect brand level demands. If generic advertising is differentially influencing the demand of individual brands, the more a brand is benefiting from it, the less likely it is to invest in brand advertising. It is therefore of importance to investigate the implications of brand level demand response to generic and brand advertising for optimal brand advertising policies. Clearly, this is an empirical matter. Unfortunately much empirical work on the effectiveness of generic and brand advertising have used aggregated demand models which do not account for consumers' preferences and competition among brands.

Previous studies that have investigated the optimal allocation of generic advertising over time have not incorporated brand advertising in the demand function. However omitting brand advertising when it has an impact on demand may bias estimated effect of generic advertising and thereby the optimal generic advertising policies. Another question worth investigating is: what are the implications of the double impact of brand and generic advertising on demand for optimal generic advertising policies?

The purpose of this research is to provide empirical evidence on the optimality of brand and generic advertising in a differentiated product oligopoly. We first assess how brand and generic advertising affect brand level demand for fluid milk and whether generic advertising has a differential effect on individual brands. To do this, we develop and estimate a nested logit demand system wherein generic advertising coefficients are brand-specific. This model is similar to Dube et al. (2005) in that we model the long-term effect of advertising using goodwill stocks; however, our model incorporates both brand and generic advertising, is estimated at the supermarket chain level, and is analyzed in a Bayesian framework.

Next the estimated demand relationships are used to determine whether observed brand advertising is optimal. A subgoal is to determine whether manufacturers should adopt a sporadic advertising strategy or advertise more frequently. To this end we develop a dynamic model of the vertical relationship between manufacturers and retailers wherein retailers act as vertically integrated firms with respect to their private label brands and play a Nash-Bertrand game in the retail market[1]. Manufacturers are Stackelberg leaders in the vertical market. They compete in wholesale price and brand advertising in the wholesale market and base their decisions on the observed state variables which consist of brand and generic advertising goodwill stocks. We characterize the Markov Perfect Equilibrium of this game under the estimated demand parameters and an exogenous allocation (to the brand manufacturers) of generic advertising. It is important to note that brand manufacturers do not choose the level of generic advertising in our model.

Lastly, we analyze the impact of product differentiation and long run effects of both generic and brand advertising for optimal generic advertising policy. To accomplish this, we

[1] Cohen and Cotterill (2008) test alternative structural pricing games for fluid milk brands in supermarket chains in Boston and report that this specification for retail pricing is the best.

formulate a dynamic optimization model where generic advertising is allocated over time so as to maximize the present value of the current and future fluid milk revenues to farmers, net of advertising costs. We solve for optimal generic advertising strategies under two brand advertising regimes: 1) a zero brand advertising regime; 2) brand manufacturers invest in brand advertising.

The estimation procedure uses Information Resources Inc. (IRI) supermarket chain level data on three fluid milk brands (Hood, Garelick, and private label) in three U.S. markets: Boston, Hartford and Providence. Demand side estimates indicate that fluid milk brand advertising is effective for increasing brand level demand, and generic advertising has a differential effect on individual brands. On the supply side, we found that (i) it is not optimal for brand manufacturers to advertise in the presence of generic advertising; (ii) brand manufacturers would gain substantially by switching to their Markov Perfect equilibrium advertising policies, which corresponds to no brand advertising, and (iii) the average predicted optimal generic advertising expenditures are significantly higher than observed generic advertising expenditures.

The rest of the paper is organized as follows. We review the related literature in section 2. In section 3 we provide a preliminary analysis of the data to motivate our modeling choices. The model is presented in section 4. Section 5 discusses demand and supply side estimation procedures. Empirical results are presented in section 6. Section 7 concludes.

2. Related literature

This study is closely related to research from two areas: literature on optimal advertising strategies in a dynamic oligopoly framework and commodity promotion literature on the effectiveness of generic advertising. We discuss relevant works from each of these literatures in turn.

Optimal advertising policies in dynamic oligopolies

Several studies in the marketing and industrial organization literatures have addressed the optimal allocation of advertising over time using a dynamic oligopoly framework. One group of studies uses differential game methods (see for example Sorger 1989, Chintagunta and Vilcassim 1992, Chintagunta 1993, Erickson 1992 and 1995, Chintagunta and Jain 1995, Espinosa and Mariel 2001, Prasad and Sethi 2004, and Bass et al. 2005). Of all these studies, only Bass et al. analyze the separate and dynamic effects of brand and generic advertising on sales. However, they provide no empirical framework of evidence concerning how much brand and generic advertising is optimal. Moreover, brand owners are assumed to choose the levels of both brand and generic advertising in their model whereas in our study the allocation of generic advertising over time is exogenous to brand manufacturers. Although the differential game approach has the advantage of providing analytic solutions for the equilibrium advertising strategies, its main drawbacks are that it cannot accommodate more sophisticated demand models, and, advertising is the only marketing instrument available to firms. Our study has three strategic variables: generic advertising, brand advertising, and prices. The latter two are under the brand manager control.

Another group of studies considers more complex demand systems and rely on numerical dynamic programming methods to solve for equilibrium advertising strategies (see for example Dube et al. 2005, Sriram and Kalwani 2005, and Tan 2004). This study is closely related to Dube et al. in that it investigates optimal advertising when it has a long term effect on brands' demand, using a two-step estimation method. A discrete choice model incorporating advertising goodwill stock is specified and estimated without imposing any supply-side restriction. Demand parameters estimates are then used to solve a dynamic game for optimal advertising strategies.

However, two important features distinguish this work from Dube et al. They specify a homogeneous logit demand system incorporating brand advertising which they estimate using a partial maximum likelihood procedure. Moreover, their analysis is at the market level and does not account for the vertical relationship between manufacturers and retailers. Our work innovates in three ways. First, we specify a nested logit demand model that incorporates both generic and brand advertising goodwill stocks; we then analyze the model in a Bayesian framework, thus avoiding the computational challenge of estimating a demand system that requires calculating high-order integrals. Second, this is the first empirical study to analyze the optimality of generic and brand advertising using a dynamic oligopoly framework. Finally, this is the first study of optimal pricing and advertising that recognizes food manufacturers sell product through retailers and one thus needs to account for the vertical pricing relationship between manufacturers and retailers.

Effectiveness of generic advertising at increasing demand

Numerous studies have investigated the effectiveness of generic advertising for various commodities including fluid milk (Kinnuncan 1986 and 1987, Forker and Liu 1989, Ward and Dixon 1989, Kamp and Kaiser 1999, Kaiser and Liu 1998), yogurt (Hall and Forker 1983), cheese (Kinnuncan and Fearon 1986, Blaylock and Blizard 1988), meat (Brester and Schroeder 1995), prunes (Alston et al. 1998, Crepsi and Marette 2002), citrus (Lee and Brown 1992), and potatoes (Jones and Ward 1989). The vast majority of these studies have estimated an aggregate demand model with a distributed lag specification for advertising. Most of these studies find that generic advertising has a positive lagged effect on demand.

A few studies have looked at the combined effects of generic and brand advertising on demand (Kaiser and Liu 1998; Hall and Forker 1982, Kinnuncan and Fearon 1986, Lee and

Brown 1992, Brester and Schroeder 1995). However, none of these studies investigate how the generic advertising manager and individual brand managers should optimally advertise over time. Hall and Forker found that brand advertising is more effective than generic advertising in expanding demand for yogurt in California. Kaiser and Liu showed that the generic advertising elasticity of demand is larger than its brand counterpart for cheese. However for fluid milk, using two different demand specifications, they found conflicting results. Using a nonlinear Rotterdam model incorporating advertising effects, Brester and Schroeder found that branded beef, pork, and poultry advertising elasticities are significantly different from zero whereas generic beef and pork advertising elasticities are not. Only one study has analyzed the effects of generic and brand advertising on demand at the brand level. Using a two-choice conditional logit model incorporating only contemporaneous advertising effects, Crespi and Marette (2002) found that prune brand advertising enhances the differentiation of competing brands whereas generic advertising does the opposite.

Kaiser and Liu (1998) used the estimates of their aggregate demand equation to simulate the effects of different allocations of funds between generic advertising and brand advertising. However, they did not allow for competition, nor did they solve for the optimal generic and brand advertising expenditures. Crespsi and Marette (2002) analytically investigated the effect of an increase of generic advertising on brands profits using a static three-stage model wherein the marketing board first chooses the generic advertising assessment fee to maximize industry profits. Brands owners then choose their brand advertising expenditures, and lastly compete in price. Kamp and Kaiser (2000) investigated the implications of asymmetric response to generic advertising for optimal temporal allocation of generic advertising of fluid milk in New York City. However their demand model does not incorporate product differentiation and brand advertising. Ignoring product differentiation and brand advertising can bias the estimated effect of generic advertising on demand and thereby the optimal allocation of advertising since optimal advertising depends on the relationship between advertising and demand. Moreover, they only consider the optimal allocation of advertising over time, assuming that total advertising expenditures are maintained at fixed levels.

This study differs from previous studies on the effectiveness of generic advertising and optimal allocation of generic advertising in many aspects. First, it explicitly models the demand side using a discrete choice model wherein both generic and brand advertising have long run effects on the utility consumers derive from choosing a specific brand. In our model generic advertising does not just increase the total market demand; it is also assumed to alter brand preferences (Chakravarti and Janiszewski 2004, Crespsi and Marette 2002). Second, this study models retail prices, wholesale prices and brand advertising as arising from a dynamic vertical game with brand managers choosing wholesale brand prices and brand advertising, and retailers choosing retail brand prices. Previous studies' emphasis is on the demand side effect of advertising. Here, our main objective is to investigate the implications of estimated demand for optimal levels of advertising. Finally, unlike Kamp and Kaiser (2000), this study focuses on the optimal level of generic advertising expenditures, not its optimal temporal allocation given a fixed level of advertising expenditures.

3. Preliminary analysis of the data

We motivate our modeling choices in the next sections by some graphical analysis of pricing, generic and brand advertising.

The data used in this study were obtained from the Food Marketing Policy Center at the University of Connecticut and covers the period February 1996 until July 2000. It consists of dollar sales, volume sales, and advertising expenditures for three fluid milk brands (Hood, Garelick, and private label) across IRI supermarket chains in Boston, Hartford and Providence.

Volume sales measured by number of gallons and dollar sales data are four-week interval observations. An average price per gallon variable is obtained by dividing dollar sales by volume sales converted into number of gallons. This variable measures the average price consumers pay for a gallon of milk and incorporates any price reduction.

Advertising data consists of weekly brand advertising expenditures (spot television), weekly local and national generic advertising expenditures (network television, cable television, and syndication television), and quarterly national print media generic advertising expenditures. The data have no available brand advertising for private label; however, there is no brand advertising for private label. Supermarkets "advertise" private label brand of milk only when they put them on promotion; such advertising conveys price information rather than brand differentiation.

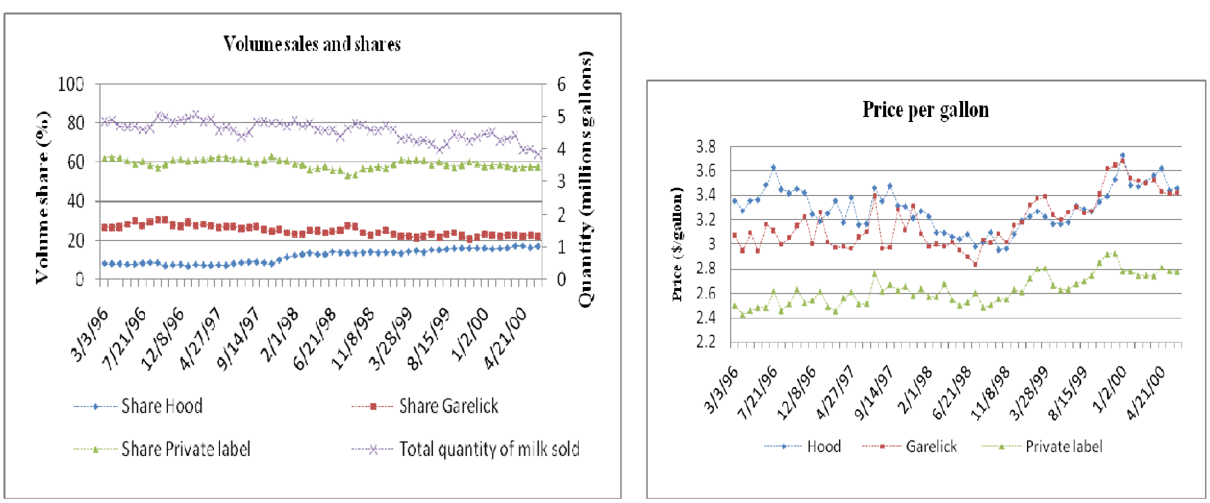
In order to match price and quantity data, weekly brand advertising data were aggregated and quarterly print media generic advertising data were interpolated. Moreover, following Wang et al. 2004 and Blisard et al. 1999, it is assumed that national advertising has uniform impacts on individuals and to obtain the advertising in print media in a market, national advertising expenditures are multiplied the by the proportion of local population in national population. Total generic advertising expenditure in each market is then computed as the sum of deflated television and print media generic advertising expenditures.

Pricing and volume sales

The left panel of figure 1 shows total quantity and volume shares of milk by brand in Boston. Total milk consumption is decreasing over time. Private label is the leading brand in this market, followed by Garelick and Hood. However, the volume share of private label and Garelick declines over time whereas that of Hood increases significantly, from 8% in March 1996 to about 17% in July 2000. Ignoring price and brand advertising for the moment, these trend patterns raise important questions about fluid milk generic advertising. How effective is generic advertising in increasing fluid milk demand? Does it have different effects on each brand of milk? If the primary goal of generic advertising is to increase demand without favoring any particular brand, then abstracting from price impacts and brand advertising, it is not working in Boston. Clearly, one must consider pricing and brand advertising when analyzing generic advertising impacts.

The right panel of the figure displays the average price of a gallon of milk by brand in Boston. Prices of the three brands have an upward trend. Also note that the price gap between Hood and Garelick narrows over time. Although not as visible, the price gap between Hood and Private label also narrows over time.

Figure 1 Average prices and volume shares of fluid milk in Boston



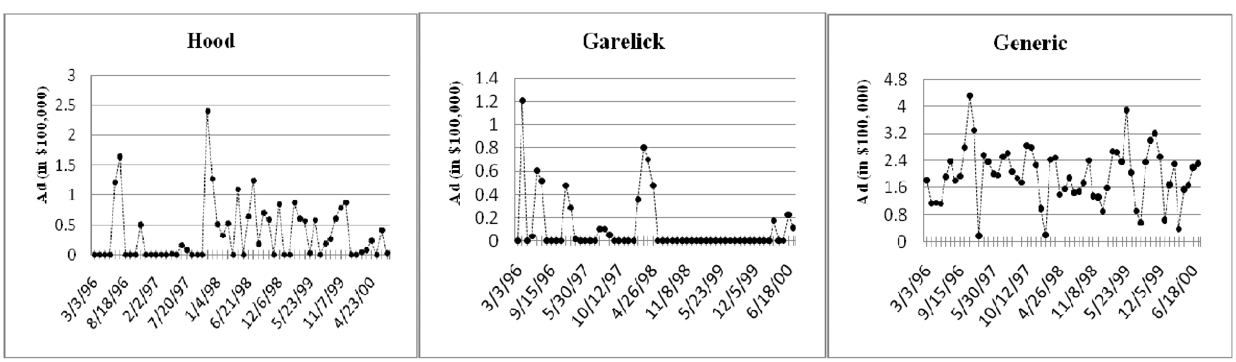
Brand and generic advertising

Figure 2 displays brand and generic advertising expenditures in Boston. Note that Hood and Garelick manufacturers do not maintain a positive level of advertising every period. Periods with high level of advertising tend to be followed by periods with low or zero advertising. This practice appears to be more prevalent for Garelick brand than for Hood brand. Also, note that Hood and Garelick advertising are not contemporaneous. In fact, the computed correlation coefficient between Hood and Garelick advertising (controlling for city fixed effects) is 0.08. One might think that this documents the lack of coordination between the Garelick and Hood firms; however it may suggests the opposite, each avoiding the other and taking its turn in a fashion that is similar to promotions by Coke and Pepsi in the soft drink industry.

As with brand advertising, generic advertising expenditures are not constant over time but unlike brand advertising, there is no period with zero advertising. Also, generic advertising appears to be more substantial than brand advertising.

The plots of brand and generic advertising in Hartford and Providence show similar patterns.

Figure 2 Milk advertising in Boston



In summary, the preliminary analysis of the data indicates the following facts:

- Total milk consumption in Boston has decreased over time;
- The volume shares of Garelick and private label, the two leading brands, decline over time whereas that of Hood expands significantly;
- Parallel to the increase in Hood volume share is a pattern of more frequent and substantial Hood advertising, and a narrowing gap between Hood and Garelick prices and between Hood and private label prices;
- Garelick advertising is more sporadic and not as important as Hood's;
- Generic advertising is more substantial and less volatile than brand advertising;
- Hood's relative advertising and price positions seem to explain its share gain during this 1996-2000 time period.

In the following sections we develop and estimate a model that seeks to explain these stylized facts and assess the optimality of observed advertising strategies. Some of the questions we seek to answer are: what role does Hood advertising and pricing play in the expansion of Hood demand? What are the effects of generic advertising on the demand of different brands?

The graphical analysis provides guidance for modeling the effect of brand advertising on fluid milk demand and the optimal levels of brand advertising. First, a model that assumes continuous and/or uniform brand advertising strategies is not adequate for describing advertising in the fluid milk market in Boston, Hartford and Providence. Second, because advertising pulses over time, it might have a lagged effect on demand.

4. Model

The demand model

Brand and generic advertising

Brand and generic advertising are assumed to have long-run effects on demand. This intertemporal effect is captured through brand and generic goodwill stocks. Following Dube et al. (2005), each brand manufacturer uses brand advertising A^b_{jt} to increase the beginning of period brand goodwill stock and create the augmented goodwill stock

$$g^{ba}_{jt} = g^b_{jt} + \psi(A^b_{jt}) \tag{1}$$

where ψ is a non-linear and non-decreasing goodwill production function satisfying $\psi(0) = 0$.

Augmented goodwill depreciates stochastically from one period to the next as follows:

$$g^b_{j,t+1} = \lambda_b g^{ba}_{jt} + v^b_{j,t+1} = \lambda_b (g^b_{jt} + \psi(A^b_{jt})) + v^b_{j,t+1} \tag{2}$$

where $0 < \lambda_b < 1$ and $v^b_{j,t}$ are stochastic shocks assumed to be independent and identically distributed (i.i.d) across time. λ_b is the depreciation rate of brand goodwill. $\lambda_b = 0$ means that brand advertising has no carry-over effect.

Expanding (2) and combining with (1) gives

$$g^{ba}_{jt} = g^b_{jt} + \psi(A^b_{jt}) = \sum_{k=0}^{t-1} \lambda_b{}^k \psi(A^b_{j,t-k}) + \lambda_b{}^t g^{ba}_{j0} + \sum_{k=0}^{t-1} \lambda_b{}^k v^b_{j,t-k} \tag{3}$$

Now let's model generic advertising. Let g^g_t denotes the generic advertising goodwill stock in period t. we assume that

$$g^g_{t+1} = \lambda_g (g^g_t + \psi(A^g_t)) + v^g_{t+1} \tag{2'}$$

where $0 < \lambda^g < 1$, and v^g_t are i.i.d across period.

Expanding (2)'

$$g_t^{ga} = \sum_{k=0}^{t-1} \lambda_g{}^k \psi(A_{t-k}^g) + \lambda_g{}^t g_0^{gla} + \sum_{k=0}^{t-1} \lambda_g{}^k v_{t-k}^g \qquad (3)'$$

The demand model

A nested logit model is used. We modify the standard model to allow brand and generic advertising goodwill stocks to shift demand. Demand is formulated as follows. Consumers are assumed to have a two-stage decision tree process: they first decide between purchasing milk from retailer r (r =1,…,K) or not to purchase milk and then given the choice of retailer, which of the J brand of milk to purchase. This results in a nested model with K+1 exhaustive and mutually exclusive sets with the outside good the only member of group 0.

The utility consumer i derives from consuming brand j in period t is given by:

$$U_{ijkt} = \delta_{jkt} + \varsigma_{ikt} + (1-\sigma)\varepsilon_{ijkt} \qquad (4)$$

where ς_{ikt} represents the common taste for product sold by retailer k, has a distribution that depends on σ, $0 \le \sigma \le 1$, and is such that $\varsigma_{ikt} + (1-\sigma)\varepsilon_{ijkt}$ has an extreme value distribution. The parameter σ measures the degree of heterogeneity among groups (retailers); if $\sigma = 1$, the correlation among retailers goes to one and retailers are regarded as perfect substitutes. On the other hand as σ tends to zero, the correlation among retailers goes to zero. When $\sigma = 0$, the model reduces to the ordinary logit model where all the brands belong to the same group and the elasticities of substitution are perfectly symmetric.

The mean utility of brand j at retailer k, δ_{jkt}, is

$$\delta_{jkt} = \alpha_{jk} + \beta p_{jkt} + d_t + \rho\, x_{jkt} + \gamma \log(1 + g_{jt}^{ba}) + \theta_j^g \log(1 + g_t^{ga}) + \xi_{jkt} \qquad (5)$$

In (5) α_{jk} is consumers' intrinsic preference for band j sold at retailer k, d_t are quarterly dummies, β is the price sensitivity parameter. x_{jkt} represents other product characteristics. g_{jt}^{ba}, and g_t^{ga} are respectively brand and generic augmented advertising goodwill stocks defined previously. ξ_{jkt} is the temporal utility shock that is observed by the consumer but not the researcher and is common to all consumers in a market.

Both brand and generic advertising augmented goodwill stocks enter the utility logarithmically to provide a well behaved objective function and advertising optimization problem for each brand manufacturer (see Dube et al. 2005). We postulate that the effect of generic advertising on each brand is different.

The mean utility of the no purchase option is normalized to 0 and the corresponding utility is $U_{0t} = \varsigma_{0t}$.

The market share of brand j of milk sold at retailer k is given by

$$s_{j/k} = \frac{e^{\delta_{jkt}/(1-\sigma)}}{D_k},$$

where

$$D_k = \sum_{j \in \Im_k} e^{\delta_{jt}/(1-\sigma)}$$

$\Im_k$ is the set of products sold by retailer k.

A consumer chooses retailer k with probability

$$s_k = \frac{D_k^{(1-\sigma)}}{\sum_k D_k^{(1-\sigma)}}$$

The sales of brand j at retailer k expressed as a share of milk sold at all retailers is the product its share of that retailer's milk sales times that retailer's share of milk sales in the total market, that is

$$s_{jkt} = s_{j/k}\, s_k = \frac{e^{\delta_{jkt}/(1-\sigma)}}{D_k^{\sigma}\left[\sum_k D_k^{(1-\sigma)}\right]} \tag{6}$$

Suppose that the market size M is fixed. This does not eliminate the supposed market-size effect of generic advertising since an increase in the total quantity of products sold will result in a decrease in the share of the outside good.

Based on above market shares, the expected demand for brand j at period t in retailer k given price p_{jkt}, brand and generic goodwill stocks $g_t^{ba} = [g_{jt}^{ba}, \cdots, g_{Jt}^{ba}]$ and g_t^{ga}, the levels brand and generic advertising $A_t^b = [A_{1t}^b, \cdots A_{Jt}^b]$ and A_t^g, and the unobserved demand characteristics $\xi_t = [\xi_{1t}, \cdots, \xi_{Jt}]$ is

$$D_{jt}(p_t, A_t^b, A_t^g, g_t^{ba}, g_t^{ga}, \xi_t) = M * s_{jt} \tag{7}$$

The supply model

Profit functions and evolution of the state variables

Denote c_{mj} and c_{rj} manufacturers' and retailers' marginal costs. Assuming prices and advertising decisions are made prior to the realization of the demand shocks ξ_{jt} (as in Dube el al. 2005 and Sriram and Kalwani 2005), players base their decisions on expected profits. The expected profit of manufacturer m in period t is given by

$$\pi_{mt}(p_t^w, p_t, A_t^b, A_t^{gf}, g_t^b, g_t^g) = \int\left[M \sum_{j\in\Im_m}(p_{jt}^w - c_{mj}) s_{jt}(p_t(p_t^w), A_t^b, A_t^g, g_t^b, g_t^g, \xi)\right]\mathrm{prob}(\xi)d\xi - \sum_{j\in\Im_m} A_{jt}^b \tag{8}$$

where $\mathrm{prob}(\xi)$ is the probability distribution function of the demand shocks ξ.

Likewise, the expected profit of retailer k in period t is

$$\pi_{kt}(p_t^w, p_t, A_t^b, A_t^{gf}, g_t^b, g_t^g) = \int M \sum_{j\in\Im_k}(p_{jt} - p_{jt}^w - c_{rj}) s_{jt}(p_t, A_t^b, A_t^g, g_t^b, g_t^g, \xi) prob(\xi) d\xi$$

As mentioned previously, generic advertising for fluid milk in each of the markets under consideration comes from three sources: local television advertising, national television advertising, and national print media. Since in each market generic advertising managers have no direct control on national generic advertising funds, we analyze the optimal level of local television generic advertising given a fixed level of national television print media advertising expenditure. We assume that the generic advertising manager bases his decision on the revenue net of advertising costs which accrues to farmers, given by

$$\pi_t^{gf} = p_t^f \sum_{j=1}^{J} q_{jt}(p_t, A_t^b, A_t^g, g_t^b, g_t^g) - A_t^g \tag{9}$$

where p_t^f and q_{jt} are farm level price of fluid milk and brand j fluid milk demand in period t, respectively.

There are three states variables in the model. The first two state variables, $g_{jt}^b, j = 1,2$, are brand advertising goodwill stocks. They evolve as a Markov process

$$g_{j,t+1}^b = \lambda_b(g_{j,t}^b + \psi(A_{jt}^b)) + v_{j,t+1}^b,$$

where the v_{jt}^b are random shocks to brand goodwill depreciation and are i.i.d $N(0, \sigma_{vb}^2)$.

The third state variable, g_t^g, is the generic advertising goodwill stock; it also evolves as a Markov process

$$g_{t+1}^g = \lambda_g(g_t^g + \psi(A_t^g)) + v_{t+1}^g$$

where the v_t^g are random shocks to generic goodwill depreciation and are i.i.d $N(0, \sigma_{vg}^2)$.

A vertical dynamic price and advertising game between manufacturers and retailers

We model the vertical relationship between manufacturers and retailers, taking into account the dynamics in wholesale prices and brand advertising decisions. The game is described as follows.

At the beginning of each period, the state vector (g_t^b, g_t^g) is observed by all manufacturers. Based on the observed state, each brand manufacturer makes his marketing decision $\sigma_m(g_t^b, g_t^g) = (p_t^w, A_t^b)$, and each retailer chooses its prices $\sigma_k(p_t^w) = (p_{kt})$. This restriction of manufacturer decisions to depend only on the current goodwill is because the state vector (g_t^b, g_t^g) contains all necessary information required to forecast current and future sales. Once the state vector has been realized and firms have made their decisions, the demand shocks ξ_t are realized and the players receive their current profits.

Each brand manufacturer maximizes its discounted present value by choosing its wholesale prices path p_{jt}^w and brand advertising spending path A_{jt}^b. The Bellman equation for manufacturer m is given by:

$$V_m(g_t^b, g_t^g) = \max_{\{p_{jt}^w, A_{jt}^b\}} \{\pi_{mt}(p_t^w, p_t, A_t^b, A_t^g, g_t^{ba}, g_t^{ga}) + \delta E[V_m(g_{t+1}^b, g_{t+1}^g)]\}, \tag{10}$$

where δ is the discount factor.

We assume that retailers behave as followers in a manufacturer Stackelberg game when setting retail prices of manufacturer brands. Retailers make price decisions on their own private label brands. Several other vertical pricing models could have been considered and tested against each other but the model used appears more realistic for the milk market studied. There is evidence that milk processors are Stackelberg leaders for their brands and at least one supermarket chain, specifically Stop & Shop, has integrated private label operations. We further assume that retailers are myopic, that is they do not take into account the effect of their price decisions on future profits; this assumption is reasonable since there is no intertemporal link in demand from the retailers' perception. Each retailer then chooses the retail price of its private label (its wholesale price is equal to the processing marginal cost) and the retail prices of manufacturer-owned brands to maximize its profit in each period, given manufacturers' choices of wholesale prices and advertising for brands.

The solution concept used in the brand advertising game is the Markov Perfect Nash Equilibrium as in many empirical dynamic games (e.g. Dube et al. 2005, Sriram and Kalwani 2005, Tan 2004, Doraszelski and Markovich 2007).

Generic advertising manager's dynamic programming problem

We assume that the role of the generic advertising manager is to choose the level of generic advertising in each period to maximize the farmers' current and future revenue, net of advertising cost. Given the generic and brand goodwill stocks g_t^g and g_t^b, the manager chooses generic advertising path $\{A_t^{gf}\}_{t=1}^{\infty}$. His objective function then satisfies the Bellman equation

$$V_{gf}(g_t^b, g_t^g, s_t) = \max_{\{A_t^{gf}\}} \{\pi_t^{gf}(A_t^{gf}, g_t^b, g_t^{ga}) + \delta E[V_{gf}(g_{t+1}^b, g_{t+1}^g, s_{t+1})]\} \tag{11}$$

5. Estimation routines

Demand estimation

The objective here is to estimate the parameters in the demand function (6) without imposing any supply-side restriction.

A logarithmic transformation of the market share equations gives the following demand equation:

$$y_{jkt} \equiv \ln(\frac{s_{jkt}}{s_{0t}}) = \alpha_{jk} + d_t + \beta p_{jkt} + \rho x_{jkt} + \gamma \log\left(1 + \psi(A_{jt}^b) + g_{jt}^b\right) + \theta_j^g \log\left(1 + \psi(A_t^g) + g_t^g\right) + \sigma \ln(s_{j/k}) + \xi_{jkt} \tag{12}$$

where $g_{jt}^b = \sum_{h=1}^{t-1} \lambda_b{}^h \psi(A_{j,t-h}^b) + \lambda_b{}^t g_{j0}^{ba} + \sum_{h=0}^{t-1} \lambda_b{}^h v_{j,t-h}^b$, $g_t^g = \sum_{h=1}^{t-1} \lambda_g{}^h \psi(A_{t-h}^g) + \lambda_g{}^t g_0^{ga} + \sum_{h=0}^{t-1} \lambda_g{}^h v_{t-h}^g$,

Following Dube el al. (2005), we specify the brand advertising production function as

$$\psi(A) = \begin{cases} \log(1 + A - G) & \text{if } A \geq G, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

where G is a threshold parameter to be estimated. This threshold production function implies that advertising is ineffective up to a level G, and beyond this level G advertising has diminishing returns. If G = 0, then we have a strictly concave advertising response function.

The identification of the threshold parameter G is likely to be a problem if there is not enough exogenous variations in the advertising data. But Figure 2 above shows that there are considerable variations in brand advertising; periods with small advertising tend to follow periods with high advertising and small advertising levels occur frequently in our data. According to Dube et al., these features of the data should be able to identify G.

For generic advertising production function, we use a simple concave sales response function ($\psi(A) = \log(1 + A)$) because generic advertising data has only few periods with very low values; this low exogenous variation cannot identify a threshold model.

Three remarks can be made about equation (12). Firstly, the transformed market share variable y_{jkt} is a function of the history of goodwill depreciation shocks and observed advertising levels until period t. Secondly, prices, advertising, and conditional market shares are potentially endogenous. Thirdly, equation (12) is a nonlinear equation where the nonlinear part includes both model parameters and the random terms v_{jt}^{b}, v_{t}^{g}, g_{j0}^{ba}, and g_{0}^{ga}. As a result, we cannot write the conditional expectation of y_{jkt} as

$\alpha_{jk} + d_t + \beta p_{jkt} + \rho x_{jkt} + \gamma \log\left(1 + \sum_{h=0}^{t-1} \lambda_b{}^h \psi(A_{j,t-h}^{b})\right) + \theta_j^g \log\left(1 + \sum_{h=0}^{t-1} \lambda_g{}^h \psi(A_{t-h}^{g})\right) + \sigma \ln(s_{j/k})$. Because of this, we cannot use a linear instrumental variable technique. A maximum likelihood based approach is required. A full maximum likelihood estimation requires specifying the joint distribution of the whole history of observations $y_{jk} = (y_{jk1},..., y_{jkT})$, and would require calculation of integrals of order T (one must integrate over the whole history of goodwill depreciation shocks v_{jt}^{b} and v_{t}^{g}). To avoid such high-order integral computation, Dube et al. (2005) used a partial maximum likelihood estimator. However the properties of this estimator like those of the full maximum likelihood estimator rely on asymptotic theory (large sample size). Given our relatively small sample size, we will use a Bayesian approach instead. It does not require integrating out the whole history of goodwill depreciation shocks v_{jt}^{b} and v_{t}^{g}. Rather it considers these random shocks as additional model parameters.

To account for the endogeneity of prices and within retailer brand shares, we use instrumental variables and specify the joint distribution of unobserved demand shocks, prices, and within retailer brand shares (Villas-Boas and Winer 1999). As instrumental variables for prices we use input prices (price of raw milk, price of electricity, price of diesel) interacted with brand dummies (Villas-Boas 2007). We specify the following price equation:

$$p_{jkt} = \theta_j Z_{jkt} + \eta_{jkt}$$

where Z_{jkt} is the vector of instruments and η_{jkt} a is random variable normally distributed and correlated with the demand shock ξ_{jkt}.

As instruments for the within retailer brand shares, we use prices of other brands sold by the same retailer (as Berry 1994 suggested) and retailers' fixed effects. The following within retailer brand share equation is specified:

$$\ln(s_{j/k}) = \tau_j W_{jkt} + \varsigma_{jkt}$$

where W_{jkt} is the vector of instruments and ς_{jkt} a is random variable normally distributed and correlated with the demand shock ξ_{jkt}.

Likelihood function

Define $g_0^{ba}=(g_{10}^{ba},\cdots,g_{J0}^{ba})$, $\Theta=(\{\alpha_{jk}\},\beta,\gamma,\{\theta_l^g\},\lambda_b,\lambda_g,G,\sigma)$, $X_{jkt}=(1,p_{jkt},x_{jkt})$, $\varphi_j=(\alpha_j,\beta,\rho)$, and

$$m_{jkt}=X_{jkt}\varphi_j+\gamma\log\left(1+\sum_{\tau=0}^{t-1}\lambda^{b\tau}\psi(A_{jk,t-\tau}^b)+\lambda^{bt}g_{jk0}^{ba}+\sum_{\tau=0}^{t-1}\lambda^{b\tau}v_{jk,t-\tau}^b\right)$$
$$+\theta_j^g\left(\sum_{\tau=0}^{t-1}\lambda^{gf\tau}\psi(A_{kt-\tau}^{gf})+\lambda^{gft}g_{k0}^{gla}+\sum_{\tau=0}^{t-1}\lambda^{gf\tau}v_{k,t-\tau}^{gf}\right)+\sigma\ln(s_{j/k})$$

The demand shocks ξ_{jkt} can then be written as $\xi_{jkt}=y_{jkt}-m_{jkt}$.

The conditional likelihood of observing $(\xi_{jkt},\eta_{jkt},\varsigma_{jkt})$ is

$$f(\xi_{jkt},\eta_{jkt},\varsigma_{jkt}\mid\{v_{j\tau}^b\}_{\tau=1}^t,\{v_\tau^g\}_{\tau=1}^t,g_{j0}^{ba},g_0^{ga};\Theta,\theta,\tau)$$
$$=f(\xi_{jt}\mid\eta_{jt},\varsigma_{jt},\{v_{j\tau}^b\}_{\tau=1}^t,\{v_\tau^g\}_{\tau=1}^t,g_{j0}^{ba},g_0^{ga};\Theta)f(\eta_{jt},\varsigma_{jt};\Sigma,\theta,\tau)$$

where $f(\xi_{jkt}\mid\eta_{jkt},\varsigma_{jkt},\{v_{j\tau}^b\}_{\tau=1}^t,\{v_\tau^g\}_{\tau=1}^t,g_{j0}^{ba},g_0^{ga};\Theta)$ is the conditional density distribution of the normally distributed random variable ξ_{jkt} on the normally distributed random variables η_{jkt} and ς_{jkt}, all having zero mean and covariance matrix Σ

The conditional likelihood of observing $(\xi_{jkt},\eta_{jkt},\varsigma_{jkt})$ is therefore

$$f(\xi_{jkt},\eta_{jkt},\varsigma_{jkt}\mid\{v_{j\tau}^b\}_{\tau=1}^t,\{v_\tau^g\}_{\tau=1}^t,g_{j0}^{ba},g_0^{ga};\Theta,\theta)=\frac{1}{2\pi\,|\Sigma|^{1/2}}\exp\left(-\frac{1}{2}\begin{pmatrix}\xi_{jkt}\\\eta_{jkt}\\\varsigma_{jkt}\end{pmatrix}'\Sigma^{-1}\begin{pmatrix}\xi_{jkt}\\\eta_{jkt}\\\varsigma_{jkt}\end{pmatrix}\right)$$

The joint distribution of demand $y_{jkt}=m_{jkt}+\xi_{jkt}$, price $p_{jkt}=\theta_j Z_{jkt}+\eta_{jkt}$, and logarithm of within retailer brand share $\ln(s_{j/k})=\tau_j W_{jt}+\varsigma_{jt}$, conditioning on advertising shocks and initial goodwill stocks is the given by

$$f(y_{jkt}, p_{jkt}, \ln(s_{j/k}) \mid \{v_{j\tau}^b\}_{\tau=1}^t, \{v_\tau^g\}_{\tau=1}^t, g_{j0}^{ba}, g_0^{ga}; \Theta, \theta)$$

$$= \frac{1}{2\pi |\Sigma|^{1/2}} \exp\left(-\frac{1}{2} \left(\begin{pmatrix} y_{jkt} \\ p_{jkt} \\ \ln(s_{j/k}) \end{pmatrix} - \begin{pmatrix} m_{jkt} \\ \theta_j Z_{jkt} \\ \tau_j W_{jkt} \end{pmatrix} \right)' \Sigma^{-1} \left(\begin{pmatrix} y_{jkt} \\ p_{jkt} \\ \ln(s_{j/k}) \end{pmatrix} - \begin{pmatrix} m_{jkt} \\ \theta_j Z_{jkt} \\ \tau_j W_{jkt} \end{pmatrix} \right) \right).$$

The conditional likelihood of observing $(y, p, \{\ln(s_{j/k})\})$ is then:

$$f(y, p, \{\ln(s_{j/k})\} \mid \{v_\tau^b\}_{\tau=1}^T, \{v_\tau^g\}_{\tau=1}^T, g_0^{ba}, g_0^{ga}; \Theta, \theta, \tau)$$

$$= \prod_{k=1}^K \prod_{j=1}^J \prod_{t=1}^T f(y_{jkt}, p_{jkt}, \ln(s_{j/k}) \mid \{v_\tau^b\}_{\tau=1}^t, \{v_\tau^g\}_{\tau=1}^t, g_0^{ba}, g_0^{ga}; \Theta, \theta, \tau)$$

Bayesian analysis

We assume

$$v_{jt}^b \sim N(0, \sigma_{vb}^2),\ v_t^g \sim N(0, \sigma_{vg}^2),\ g_{j0}^{ba} \sim N\left(\frac{\sum_{t=1}^T \psi(A_{jt}^b)}{T(1-\lambda_b)}, \frac{1}{1-\lambda_b^2} \sigma_{vb}^2 \right),\ g_0^g \sim N\left(\frac{\sum_{t=1}^T \psi(A_t^g)}{T(1-\lambda_g)}, \frac{1}{1-\lambda_g{}^2} \sigma_{vg}^2 \right).$$

These last two assumptions follow Dube et al. (2005).

We further specify the following prior distributions:

$\varphi_j \sim N_3(0, 100 I_3)$, $\gamma \sim N(0, 100)$, $\theta_j^g \sim N(0, 100)$, $\theta_j \sim N_4(0, 100 I_4)$, $\tau_j \sim N_3(0, 100 I_3)$, $G \sim N^+(0, 100)$, $\lambda_b \sim \text{Beta}(0.8, 0.8)$, $\lambda_g \sim \text{Beta}(0.8, 0.8)$, $\sigma \sim \text{Beta}(0.8, 0.8)$, $\sigma_{vb}^{-2} \sim IG(2, 0.01)$, $\sigma_{vg}^{-2} \sim IG(2, 0.01)$, and $\Sigma \sim \text{Inverse Wishart}(10, 10 I_9)$, where I_p represents a p-dimensional identity matrix, and N_p a p-dimensional normal distribution.

The joint posterior density of all model parameters is then given by

$$f(y, p, \ln(s_{j/k}), \{v_\tau^b\}_{\tau=1}^T, \{v_\tau^g\}_{\tau=1}^T, g_0^{ba}, g_0^{ga}, \Theta, \theta, \tau) = \left(\prod_{k=1}^K \prod_{j=1}^J \prod_{t=1}^T f(y_{jkt}, p_{jkt}, \ln(s_{j/k}) \mid \{v_{jk,\tau}^b\}_{\tau=1}^t, \{v_{k\tau}^g\}_{\tau=1}^t, g_{jk0}^{ba}, g_{k0}^{ga}; \Theta, \theta, \tau) \right)$$

$$\times \left(\prod_{k=1}^K \prod_{j=1}^J \prod_{t=1}^T \pi(v_{jkt}^b \mid \sigma_{vb}^2) \right) \times \left(\prod_{k=1}^K \prod_{t=1}^T \pi(v_{kt}^g \mid \sigma_{vg}^2) \right) \times \pi(g_{j0}^{ba} \mid \lambda^b, \sigma_{vb}^2) \times \pi(g_0^{ga} \mid \lambda_g, \sigma_{vg}^2) \times \pi(\varphi_j)$$

$$\times \pi(\gamma) \times \pi(\theta_j^g) \times \pi(G) \times \pi(\lambda_b) \times \pi(\lambda_g) \times \pi(\sigma) \times \pi(\theta) \times \pi(\tau) \times \pi(\sigma_{vb}^2) \times \pi(\sigma_{vg}^2) \times \pi(\Sigma).$$

The model is estimated by a Markov Chain Monte Carlo procedure that sequentially draws from the full conditional distributions of the model parameters (see Yang et al. 2003, Gelfand and Smith 1990, Gelfand et al. 1990). Slice sampling algorithm (Neal, 2000) is used for the conditional distributions that are not of standard form.

Estimation requires the definition of an outside good. Market shares of brands under consideration are defined by converting the volume sales into servings sold, and dividing by the market size. We assume that each individual has the potential to consume 1/16th of a gallon of milk (one cup) every day; the market size is then obtained by multiplying the market population in a period by that consumption rate. The market share of the outside good is defined as the difference between one and the sum of the brands under consideration. The table below presents the summary statistics for prices per serving, market shares, and ratio of low fat to whole milk (the only other product characteristic).

Table 1: Summary statistics

		Prices ($/serving)		Market shares (%)		Low fat/whole milk ratio	
Brand	Chain	Mean	S.D.	Mean	S.D.	Mean	S.D.
Hood	Demoulas	0.176	0.006	0.629	0.213	2.039	0.240
Garelick	Demoulas	0.167	0.010	0.545	0.234	1.224	0.199
Private label	Demoulas	0.139	0.007	8.299	0.743	1.920	0.073
Hood	Shaws	0.185	0.015	0.876	0.579	7.728	9.480
Garelick	Shaws	0.172	0.013	2.408	0.428	3.315	0.495
Private label	Shaws	0.151	0.009	7.265	0.788	1.902	0.095
Hood	Star Market	0.186	0.006	1.265	0.259	2.966	0.602
Garelick	Star Market	0.176	0.011	2.064	0.233	3.392	0.483
Private label	Star Market	0.153	0.009	3.215	0.496	1.950	0.131
Hood	Stop & Shop	0.182	0.012	2.451	0.760	2.812	0.691
Garelick	Stop & Shop	0.179	0.016	2.523	0.926	2.986	0.602
Private label	Stop & Shop	0.158	0.010	12.504	2.516	1.864	0.120
Hood	Other chains	0.177	0.012	1.983	1.025	2.420	0.796
Garelick	Other chains	0.165	0.014	6.064	4.037	2.764	1.114
Private label	Other chains	0.154	0.011	9.729	5.659	1.826	0.231

Computing optimal advertising strategies

We characterize the Markov Perfect Equilibrium (MPE) of a dynamic game with two players (Hood and Garelick manufacturers) by solving the set of two simultaneous Bellman equations given by equations (10). In these equations, the unknowns are the value functions $V_m(.)$ and the optimal policies $p_j(.)$, $A_j^b(.)$, m=1,2, all of which are defined on a three-dimensional state space. We also determine optimal generic advertising policies by solving the dynamic model represented by equation (11). The state variables include brand advertising stocks and generic advertising goodwill stocks. Generic advertising goodwill stock evolves exogenously to brand manufacturers. In general, the Bellman equations do not have an analytic closed form solution and can only be solved numerically.

The carryover effect of advertising from period to period creates an inter-temporal link in demand thus making manufacturers' profits at any given period a function of previous periods advertising. This functional dependence makes it impossible to solve the manufacturer static profit function for prices without knowledge from the value function as is often the case in many empirical dynamic oligopoly models[2]. Another feature of our model is that the state variables (brand and generic advertising goodwill stocks) are continuous instead of discrete. Therefore, the numerical methods to compute Markov-perfect equilibria in dynamic oligopoly models advanced by Pakes and McGuire (1994, 2001) do not apply.

Dube et al. (2005) characterized the Markov Perfect equilibrium of their advertising game using a policy iteration algorithm. They approximated the value functions on a grid constructed by discretizing the goodwill axis. They chose approximation on a grid instead of polynomial approximation because their equilibrium advertising policies contain a kink.

We resort to the collocation method (Miranda and Fackler 2004; Judd 1988). Tan (2004) applied this method in a dynamic analysis of the U.S. cigarette market. The collocation method

[2] See for example Benkard (2004), Fershtman and Pakes (2000), Doraszelki and Markovich (2007). In these studies, the dynamic element consists of investment to improve the quality (i.e. brand specific intercepts) and therefore does not create an intertemporal (functional) link in demand.

approximates the value function by a series of "well behaved" functions, thus simplifying the dynamic programming problem to that of finding the coefficients of the basis functions.

To use the collocation method we approximate the value function for each player as a linear combination of N basis functions $\phi_1, \phi_2, \cdots, \phi_N$, whose coefficients $c_j = [c_{j1}, c_{j2}, \cdots, c_{jN}]$ are to be determined:

$$V_j(g) \approx \sum_{j=1}^{N} c_{jn}\phi_n(g)\,.$$

The basis function coefficients $c_{j1}, c_{j2}, \cdots, c_{jN}$ are fixed by requiring the approximants to satisfy the Bellman equations, not at all possible states, but rather at N collocation nodes, $g_1, g_2, \cdots, g_N$

We used Chebychev functional approximation and N=20 collocations coordinates by state dimension. Details of the steps used in solving the dynamic programming problem are provided in the appendix.

The calculations are based on a discount factor of 0.995. The marginal costs used are estimates from Dhar and Cotterill (2003).

To check for the uniqueness of equilibrium different initial guesses of the equilibrium strategies are used. One of the initial guesses corresponds to the situation where advertising manufacturers price at marginal cost, choose zero brand advertising, and earn a zero reward.

6. Estimation results

Demand model

Table 2 gives estimated results for the demand model. All the retailer-brand fixed effects are negative and significant. These retailer-brand fixed effects measure unobserved (by the econometrician) brand characteristics that vary by retailer (e.g. shelf space, shelf location or in-store coupons) and are common to all consumers shopping at a retailer; ignoring them would have an adverse effect on price and advertising elasticities (Chintagunta, Dube, and Goh 2004). The estimated fixed effects being negative and significant suggests that unobserved retailer-brand characteristics are indeed present, have a negative effect on consumer choices.

The estimated response of consumers to prices and brand advertising is common to all three brands. Own price (advertising) has a negative (positive) effect on consumers' utility. The estimated coefficient of generic advertising is positive and significant for all the three brands, and varies across brands; the estimated 95% credible intervals indicate that generic advertising has the same impact on Hood and Garelick brands but a smaller impact on private label brand.

In the demand model described above, advertising expenditure is converted into goodwill via the goodwill production function and λ_b and λ_g represent the brand and generic advertising goodwill decay rate, respectively. The estimated goodwill decay rate is 0.556 for brand advertising and 0.129 for generic advertising. These values along with their standard deviations suggest that there is evidence of carry-over effect of generic and brand advertising. In addition, brand goodwill depreciates slower than generic goodwill. It takes about a year for a unit of brand goodwill to decay to zero while a unit of generic goodwill will decay to zero in about 4 months. The carry-over feature of brand and generic advertising into future periods indicates that brand manufacturers and generic advertising manager should be forward-looking in making their advertising expenditures decisions.

Brand advertising threshold is not significantly different from zero. This contrasts with Dube et al. (2005) who found that brand advertising below 32.11 GRPs has no effect on demand for frozen entrees.

The estimated between-retailer heterogeneity parameter is 0.96, indicating that utility of consumers are highly correlated within retailer and uncorrelated between retailers, thereby rejecting the ordinary logit specification. This implies that within supermarket chain market share will get higher weight than overall market shares in the computation of own and cross price elasticities.

The variable ratio low fat to whole milk has a positive coefficient, indicating consumers' preference for low-fat milk.

Finally, the variances of brand and generic advertising goodwill shocks are close to zero, suggesting that these shocks are very small in magnitude.

Table 2 Fluid milk demand estimates

Parameters	Mean	Std Dev.	[2.5th percentile ; 97.5th percentile]
Band-store fixed effects			
Hood-Demoulas	-6.012*	0.200	[-6.366 ; -5.578]
Garelick-Demoulas	-6.291*	0.307	[-6.768 ; -5.597]
Private label-Demoulas	-3.799*	0.235	[-4.196 ; -3.289]
Hood-Shaws	-5.940*	0.213	[-6.348 ; -5.504]
Garelick-Shaws	-6.154*	0.285	[-6.599 ; -5.547]
Private label-Shaws	-3.689*	0.254	[-4.162 ; -3.162]
Hood-Star	-6.311*	0.202	[-6.714 ; -5.896]
Garelick-Star	-6.576*	0.271	[-7.019 ; -5.959]
Private label-Star	-4.149*	0.237	[-4.591 ; -3.675]
Hood-Stop&Shop	-5.394*	0.168	[-5.720 ; -5.069]
Garelick-Stop&Shop	-5.659*	0.271	[-6.058 ; -5.055]
Private label-Stop&Shop	-3. 219*	0.224	[-3.585 ; -2.702]
Hood-Other	-5.464*	0.173	[-5.838 ; -5.116]
Garelick-Other	-5.708*	0.242	[-6.062 ; -5.153]
Private label-Other	-3.297*	0.216	[-3.679 ; -2.830]
Price	-2.784*	1.354	[0.624 ; 5.813]
Brand advertising	0.017*	0.007	[0.010 ; 0.042]
Generic advertising			
Hood	1.782*	0.069	[1.712 ; 1.941]
Garelick	1.880*	0.036	[1.819 ; 1.942]
Private label	0.968*	0.030	[0.895 ; 1.017]
Brand advertising decay rate	0.556*	0.264	[0.122 ; 0.977]
Generic advertising decay rate	0.129*	0.025	[0.101 ; 0.185]
Brand advertising threshold G	8.7262	6.4492	[0.337 ; 22.647]
Share low fat/whole milk	0.005	0.004	[2.24e-4 ; 0.016]
Within retailer correlation σ	0.960*	0.031	[0.886 ; 0.989]
σ^2_{vb}	0.001*	0.00005	[7.68e-4 ; 9.75e-4]
σ^2_{vg}	0.0001*	0.00001	[9.26e-5 ; 1.40e-4]

We ran an MCMC of 25,000 iterations and used the first 15,000 iterations as burn-in.
* Significantly different from zero at the 5% level

Table 3 contains the covariance matrix for demand, prices and within retailer brand share shocks. The estimated covariances between the demand and price shocks are all close to zero, implying that prices are exogenously determined in our model. However, there exist a significant correlation between within-retailer brand shares and demand shocks, indicating that within-retailer brand shares are endogenous. Therefore, accounting for price endogeneity was not necessary but not accounting for that of within retailer share would have yielded inconsistent estimate of between-retailer heterogeneity. The exogeneity of prices is not peculiar to this study; estimating a random coefficients discrete choice model for light beer, Yang et al. (2003) found that prices are exogenous.

Table 3 Covariance matrix for demand, prices and within retailer brand shares shocks

	ξ_{Hood}	$\xi_{Garelick}$	ξ_{Pvl}	η_{Hood}	$\eta_{Garelick}$	η_{Pvl}	ζ_{Hood}	$\zeta_{Garelick}$	ζ_{Pvl}
ξ_{Hood}	1.3405* (0.2837)								
$\xi_{Garelick}$	1.1573* (0.263)	1.3809* (0.2793)							
ξ_{Pvl}	1.0239* (0.2289)	1.0352* (0.2299)	1.1353* (0.2282)						
η_{Hood}	0.0012 (0.1197)	0.0112 (0.1213)	-0.0137 (0.1105)	0.533* (0.1051)					
$\eta_{Garelick}$	-0.0003 (0.122)	0.0099 (0.1235)	-0.0154 (0.1122)	0.3488* (0.0901)	0.543* (0.1086)				
η_{Pvl}	-0.0096 (0.1216)	-0.0015 (0.123)	-0.0227 (0.1111)	0.3487* (0.0912)	0.3538* (0.0923)	0.5437* (0.1096)			
ζ_{Hood}	-0.8222* (0.3421)	-0.8996* (0.3235)	-0.8003* (0.2924)	0.2762 (0.198)	0.3131 (0.2015)	0.3235 (0.2004)	3.4472* (0.6778)		
$\zeta_{Garelick}$	-0.4362 (0.3481)	-0.2776 (0.377)	-0.4542 (0.3237)	0.116 (0.2208)	0.1001 (0.2209)	0.1024 (0.2219)	-0.515 (0.5566)	4.5909* (0.9126)	
ζ_{Pvl}	0.4755* (0.1621)	0.4384* (0.1659)	0.4707* (0.1513)	-0.0454 (0.0923)	-0.0455 (0.0944)	-0.0487 (0.0944)	-0.2881 (0.2374)	-1.1482* (0.3156)	0.8437* (0.1657)

Standard errors are given into brackets
* Significantly different from zero at the 5% level

Table 4 reports brand and generic advertising elasticities. Hood and Garelick own advertising elasticities are positive and significantly different from zero. Cross brand advertising elasticities are not significantly different from zero, meaning that Hood, Garelick, and private label brands demands are not affected by rival's brand advertising. Schmidt and Kaiser (2002), using quarterly national data and an aggregated demand model, found no effect of brand advertising on the demand of fluid milk. The explanation they gave to their results was that brand advertising aims at gaining market shares from competitors and may have no effect or negative effect on total demand. Our estimate suggests that brand advertising captures market share from the outside good instead.

Generic advertising has a positive and significant effect on Hood and Garelick demand, but a negative albeit nonsignificant effect on private label demand. This suggests that consumers respond to generic advertising message like "Got milk?" by buying a more expensive brand of milk.

Comparing Hood and Garelick demand responses to own brand advertising and generic advertising, generic advertising is far more effective at increasing these brands' demand than brand advertising. A 10% increase in Hood (Garelick) advertising increases Hood (Garelick) market share by 0.12% (0.06%) whereas increasing generic advertising by 10% increases Hood and Garelick market shares by 8.48% and 10.14%, respectively. It might then be more profitable to Hood and Garelick manufacturers to divert brand advertising expenses to generic advertising. Based on these results we anticipate that optimal Hood and Garelick advertising policies should be lower than the observed values.

Table 4 Fluid milk advertising elasticities

	Hood advertising	Garelick advertising	Generic advertising
Hood	0.012* (0.006)	-0.054 (0.062)	0.846** (0.500)
Garelick	-0.038 (0.085)	0.006* (0.003)	1.014* (0.049)
Private label	-0.038 (0.085)	-0.054 (0.062)	-0.557 (0.487)

Cell entries i, j, where i indexes row and j column, give the percent change in market share of brand i with one percent change in advertising expenditure of j.Standard deviations are given into parentheses.* , ** Significantly different from zero at the 5% and 10% level

The supply side

Now let us investigate the implications of the demand estimates for optimal levels of brand and generic advertising. Our objective is to check whether observed advertising expenditures are optimal under the estimated demand, and whether or not brand advertising is necessary.

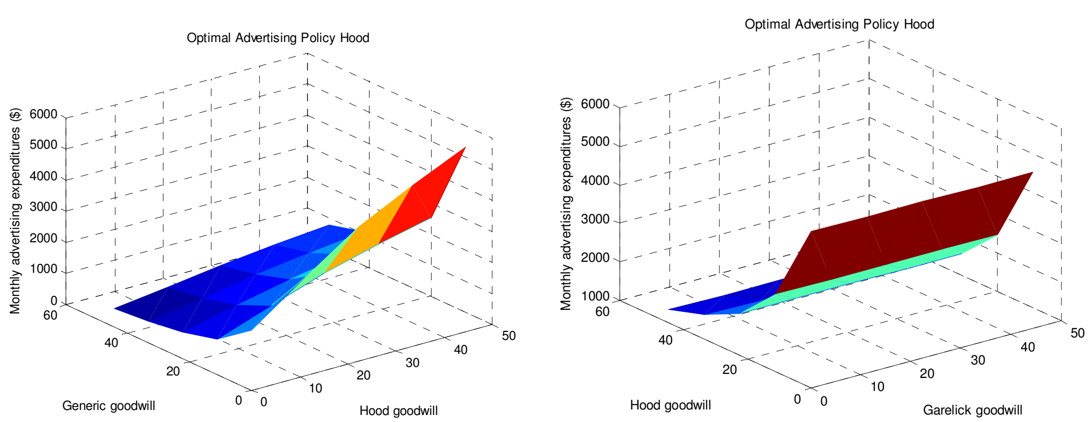
Optimal brand advertising policies

Figure 3 show optimal brand advertising policies for Hood in the Boston market as functions of brand advertising and generic advertising goodwill stocks.

First consider how Hood optimal advertising policies change with own and generic goodwill stocks. The first plot indicates that if generic advertising goodwill stock is even moderately high, it is optimal for Hood manufacturer not to advertise. Now consider how Hood optimal advertising change with own and rival's goodwill stocks. The second plot shows that Hood optimal advertising policy is not significantly affected by Garelick goodwill stock. Optimal advertising policy for Garelick shows similar pattern and therefore is not displayed.

These results imply that there is no strategic interaction between Hood and Garelick advertising policies and it is optimal for Hood and Garelick manufacturers not to advertise in the presence of generic advertising.

Figure 3 Hood optimal advertising policy as function of brand and generic goodwill stocks



We simulated 10,000 periods of the advertising game in Boston and used the last 5000 periods to computes summary statistics. Figure 4 plots 58 periods of simulated advertising for Hood and Garelick in Boston. We see from this figure that both brands have insignificant advertising expenditure in equilibrium.

Figure 4 Simulated advertising for Hood and Garelick in Boston

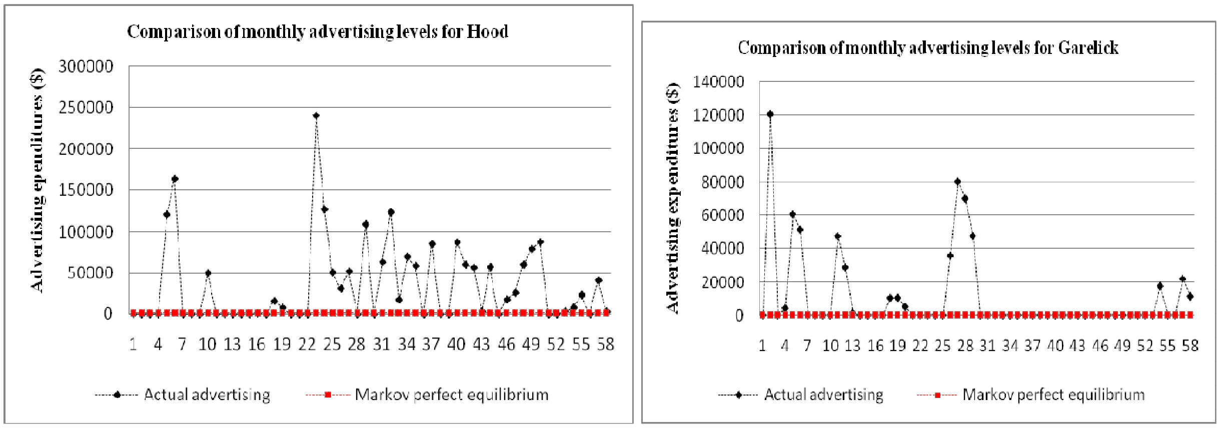


Table 5 displays average advertising expenditure and average frequency of period of zero advertising for the simulated and observed advertising levels. On average, predicted advertising levels are significantly lower than observed levels for both Hood and Garelick.

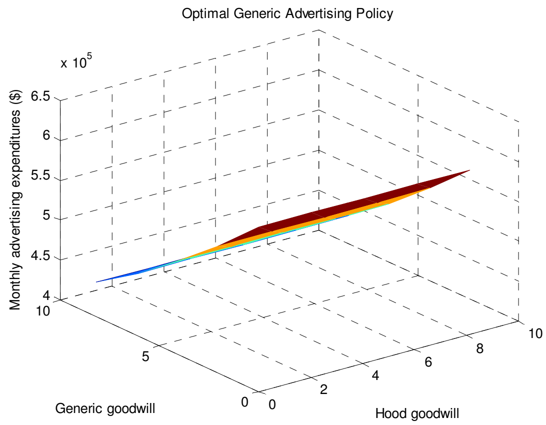
Table 5 Fluid milk advertising patterns in Boston

Advertising	Observed data		Markov Perfect Equilibrium	
	Average monthly advertising ($)	% periods with zero advertising	Average monthly advertising ($)	% periods with zero advertising
Hood	59,037	37.93	1,392	0.0
Garelick	36,815	68.96	325	0.0
Generic	195,358	0.0	364,759	

Optimal generic advertising policies

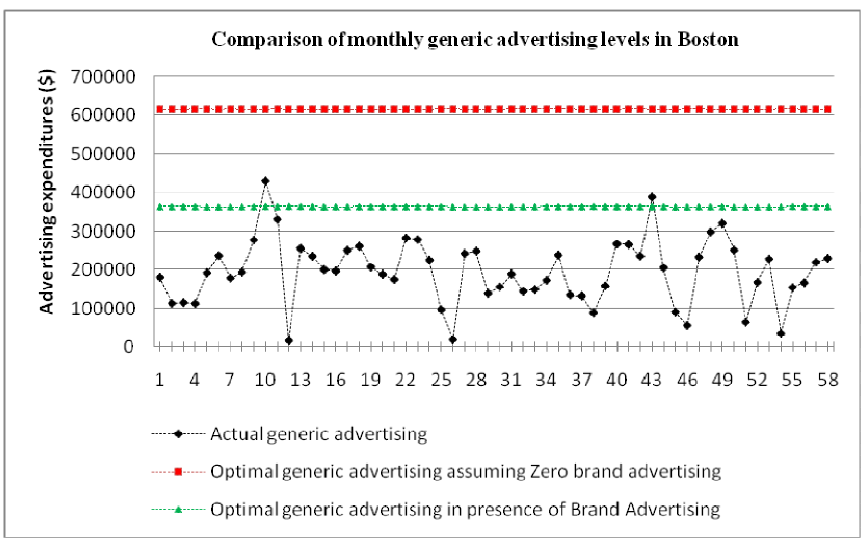
We computed optimal generic advertising policy when the state variables consist of generic and brand advertising goodwill stocks. Figures 5 plots the optimal generic advertising policy as function of generic and Hood advertising goodwill stocks in Boston. Clearly, optimal generic advertising is decreasing in generic goodwill and is not significantly affected by brand goodwill stocks.

Figure 5 Optimal generic advertising policy as function of Hood and generic goodwill stocks



We simulated optimal local generic advertising expenditures for 10000 periods in Boston and plot the last 58 periods in figure 6. This plot indicates that the optimal generic advertising policy is quite uniform; moreover, the optimal level of generic advertising is significantly lower in the presence of brand advertising than in a zero advertising regime.

Figure 6 Simulated optimal local generic advertising expenditures in Boston



Conclusions

We analyzed the optimal level of generic and brand advertising expenditures in a differentiated oligopoly market. We first developed a nested logit demand system with both generic and brand goodwill stocks as demand shifters. Demand estimates shows that both generic and brand advertising affect the demand of fluid milk but generic advertising is more effective for increasing demand than brand advertising. Advertising has a strong carry-over effect into future periods, suggesting that firms should be forward-looking when choosing their advertising expenditures.

We then investigated under the estimated demand, the optimal level of generic and brand advertising expenditures. Our results indicated that firms would increase their profit if they adopt the suggested Markov perfect Equilibrium brand advertising policies, which correspond to no brand advertising. Further, the optimal generic advertising policy does not involve pulsing, and the average predicted optimal generic advertising expenditure is significantly higher than observed generic advertising.

We estimated our demand model using Bayesian methods, allowing for price endogeneity. The advantage of this method over the Maximum Likelihood approach is that it avoids computing high order integrals, and confidence intervals of demand elasticities are obtained as byproducts of the estimation.

A limitation of this study is that we do not account for potential advertising endogeneity, due to lack of instruments; this biases the estimated relationship between demand and goodwill.

Appendix

Computing demand elasticities

Price and advertising elasticities for each city-period are computed as follows:

Let $s_{jkt} = \dfrac{e^{\delta_{jt}/(1-\sigma)}}{D_k^{\sigma}\left[\sum_k D_k^{(1-\sigma)}\right]}$,

$D_k = \sum_{j\in\Im_k} e^{\delta_{jt}/(1-\sigma)}$,

$\delta_{jt} = \alpha_j + \beta p_{jt} + d_t + \rho\, x_{jt} + \Gamma(g_{jt}^{ba}) + \theta_j^g \log(1+g_t^{ga}) + \xi_{jt}$

$\Im_k$ is the set of products sold by retailer k.

Price elasticities

$$\eta_{jlt}^{p} = \frac{\partial s_{jt}}{\partial p_{lt}}\frac{p_{lt}}{s_{jt}} = \begin{cases} -\dfrac{\beta}{1-\sigma} p_{jt}\left[1-\sigma\dfrac{e^{\delta_{jt}/(1-\sigma)}}{D_k} - (1-\sigma)s_{jl}\right] & if\ l = j \\ \beta p_{lt}\left[s_{lt} + \dfrac{\sigma}{1-\sigma}\dfrac{e^{\delta_{jt}/(1-\sigma)}}{D_k}\right] & if\ l \neq j \end{cases}$$

Brand advertising elasticities

$$\eta_{jlt}^{b} = \frac{\partial s_{jt}}{\partial A_{lt}^{b}}\frac{A_{lt}^{b}}{s_{jt}} = \begin{cases} \dfrac{\gamma}{(1+g_{jt}^{ba})(1+A_{jt}^{b}-G)}\dfrac{A_{jt}^{b}}{1-\sigma}\left[1-\sigma\dfrac{e^{\delta_{jt}/(1-\sigma)}}{D_k} - (1-\sigma)s_{jl}\right] & if\ l = j \\ -\dfrac{\gamma}{(1+g_{lt}^{ba})(1+A_{lt}^{b}-G)} A_{lt}^{b}\left[s_{lt} + \dfrac{\sigma}{1-\sigma}\dfrac{e^{\delta_{jt}/(1-\sigma)}}{D_k}\right] & if\ l \neq j \end{cases}$$

Generic advertising elasticities

$$\eta_{jt}^{g} = \frac{\partial s_{jt}}{\partial A_t^{g}}\frac{A_t^{g}}{s_{jt}} = \frac{A_t^{g}}{(1+g_t^{ga})(1+A_t^{g})}\left[\frac{\theta_j^g}{1-\sigma} - \frac{\sigma}{1-\sigma}\sum_{j\in\Im_k}\theta_j^g\frac{e^{\delta_{jt}/(1-\sigma)}}{D_k} - \sum_k\sum_{j\in\Im_k}\theta_j^g s_{jk}\right] \quad , j = 1,\ldots J$$

Numerical solution method

Step 0: Given the degree of approximation N, a set of basis functions ϕ_n, and a set of collocation nodes g_l, for each player j, guess the values of c_j of the value function approximation basis coefficients and the optimal actions σ;

Step 1: Holding the vectors of basis coefficients $\mathbf{c}_j$ and actions σ constant, solve the following system of equations:

$$\sum_{n=1}^{N} c_{jn}\phi_n(g_l) = \max_{\sigma_j}\left\{\pi_j(g_l,\sigma_j,\sigma_{-j}(g_l)) + \delta\int\sum_{n=1}^{N} c_{jn}\phi_n(h(g_l,\sigma_j,\sigma_{-j}(g_l),v))dv\right\}$$
$$= \max_{\sigma_j}\left\{\pi_j(g_l,\sigma_j,\sigma_{-j}(g_l)) + \delta\sum_{k=1}^{K}\sum_{n=1}^{N} c_{jn}\phi_n(h(g_l,\sigma_j,\sigma_{-j}(g_l),v_k))p(v_k)\right\}$$

and compute $V_j(\mathbf{c}_j,\sigma_{-j})$ for each player j. Let $\sigma_j^{'}$ denotes the vector of actions of the above maximization.

Step 2: for each player j, update the optimal actions at the collocation nodes by setting $\sigma_j \leftarrow \sigma_j^{'}$ and update the basis coefficients by using the pseudo-Newton method, which uses the following iterative update rule:

$$c_j \leftarrow c_j - [\Phi - V_j^{'}(c_j,\sigma_{-j})]^{-1}[\Phi c_j - V_j(c_j,\sigma_{-j})]$$

for each j, where Φ is the collocation matrix, whose elements are basis functions evaluated at the collocations nodes

$\Phi_{\ln} = \phi_n(g_l)$, $l = 1,\ldots,N$, $n = 1,\ldots,N$.

$V^{'}(c,\sigma)$ is the $N \times N$ Jacobian of the collocation function with respect to the basis coefficients $\mathbf{c}_j$.

Step 3: If the change in the coefficients vectors from previous iteration is less than some prescribed tolerance, stop; otherwise, return to step 1.

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