

Tian, Li and Ronald W. Cotterill. 2005. Constrained Price, Address, or Logit Brand Demand Models: An Econometric Comparison in the Boston Fluid Milk Market. *Agribusiness*, 21(2): 149-166.

APPENDIX: GENERALIZATION OF MILLS ADDRESS MODEL

We show here the derivation for a 3-good case. The utilities are stated as follows:

$$u_1 = \eta_1 + \theta - p_1 \quad (34)$$

$$u_2 = \eta_2 + \alpha\theta - p_2 \quad (35)$$

$$u_3 = \eta_3 + \beta\theta - p_3; \quad 0 < \beta < \alpha \leq 1 \quad (36)$$

θ is assumed to have a uniform distribution with pdf, $\frac{1}{b}$, over the range $[0, a]$.

η_i is a measure of unobservable product attributes or unobservable brand specific effects that cannot be accounted for with uniform distribution. For a retailer, the demand for any product is:

$$q_j = F(a \geq \theta \geq \frac{r_j - \eta_j}{\delta_j}) = \int_a^{\frac{r_j - \eta_j}{\delta_j}} \frac{1}{b} d\theta; \quad \delta_j = 1, \alpha, \beta \quad (37)$$

where r_j is the revenue a consumer pays for 1 unit of product j . The retailers inverse demand functions are derived by integrating (37) for each product:

$$r_1(q_1) = \eta_1 + a - bq_1 \quad (38)$$

$$r_2(q_2) = \eta_2 + \alpha(a - bq_2) \quad (39)$$

$$r_3(q_3) = \eta_3 + \beta(a - bq_3) \quad (40)$$

The following marginal conditions are necessary for determining the number of consumers for each product:

$$r_1(q_1) - p_1 = r_2(q_1) - p_2 \quad (41)$$

$$r_2(q_1 + q_2) - p_2 = r_3(q_1 + q_2) - p_3 \quad (42)$$

$$r_3(q_1 + q_2 + q_3) = p_3 \quad (43)$$

Condition (41) implies that the last consumer to choose product 1 or product 2 is indifferent, condition (42) implies that the last consumer to is indifferent about consuming product 2 and 3, and condition (43) implies that the last consumer has zero utility consuming product 3. When all three products are produced, condition (41) identifies the number of consumers for product 1, condition (41) and (42) identify the number of consumers for product 2, and all 3 conditions identify the number of consumers for product 3:

$$q_1 = \frac{a}{b} + \frac{\eta_1 - \eta_2}{b(1-\alpha)} - \frac{p_1 - p_2}{b(1-\alpha)} \quad (44)$$

$$q_2 = \frac{\eta_2 - \eta_3}{b(\alpha - \beta)} - \frac{\eta_1 - \eta_2}{b(1-\alpha)} + \frac{p_1 - p_2}{b(1-\alpha)} - \frac{p_2 - p_3}{b(\alpha - \beta)} \quad (45)$$

$$q_3 = \frac{\eta_3}{b\beta} - \frac{2(\eta_1 - \eta_2)}{b(1-\alpha)} + \frac{\eta_2 - \eta_3}{b(\alpha - \beta)} + \frac{p_2 - p_3}{b(\alpha - \beta)} - \frac{p_3}{b\beta} \quad (46)$$